MUTUAL EFFECT OF ELEMENTS IN A THREE-PHASE, 
TWO-ELEMENT WATTMETER 
(UDC 621.317.784.088) 
A. M. Lyubarskaya

Translated from Izmeritel'naya Tekhnika, No. 9, 
pp. 28-31, September, 1965

Ferrodynamic and electrodynamic three-phase, two-element wattmeters of high-precision classes 0.2 and 0.5 
are now being widely used. In these instruments, torque $T_q$ applied to the moving part must equal the sum of 
torques $T_{q1}$ and $T_{q2}$ produced by each element separately:

$$T_{q1} = \kappa U_1 I_1 \cos \phi_1 \quad \text{and} \quad T_{q2} = \kappa U_2 I_2 \cos \phi_2,$$

where $U$ and $I$ are the voltage and current, respectively, in the first and second elements; $\phi_1$ and $\phi_2$ are the phase 
differences between the voltage and the current in the first and second elements, respectively; $k$ is the coefficient 
of proportionality.

However, in actual instruments, owing to stray fluxes, in addition to torques $T_{q1}$ and $T_{q2}$, the moving part is 
affected by torques $D_{tz}$ and $D_{tz}$ produced by the interaction ($D_{tz}$) between the series and parallel circuits of the first 
and second elements, respectively, and the interaction ($D_{tz}$) between the series and parallel circuits of the second 
and first elements, respectively. There also exists a torque produced by the interaction between the parallel cir-
cuits of the elements, but it is so small that it can be neglected.

The interaction torques of elements are small as compared with $T_{q1}$ and $T_{q2}$; however, they can produce sub-
stantial errors if the phases between the voltages and currents differ from those which existed at the time of their 
calibration or checking.

Torques $D_{tz}$ and $D_{tz}$ play an important part in testing three-phase wattmeters by limiting the possibilities of 
their measurement with "artificial" circuits [1,2], in which the three-phase, two-element instruments are connected 
into a single-phase circuit and compared with a single-phase reference instrument.

Therefore, the evaluation of the mutual effect of the three-phase wattmeter elements, the torques produced 
by these effects in wattmeters of different designs, and the analysis of component errors in these torques for various 
connections of wattmeters are of great importance.

Let us derive expressions for the torques due to mutual effects in the elements of wattmeters connected in 
three different ways.

![Diagram](image-url)
Circuit 1 (a normal operating Aron circuit, Fig. 1a), which is also used for testing wattmeters by comparing their readings with those of two single-phase reference wattmeters.

Circuit 2 (Fig. 2a), recommended by Instruction 184-62 for testing three-phase wattmeters by comparing them with one single-phase wattmeter.

Circuit 3 (Fig. 3a), which was used at the VNIIGK for testing three-phase high-precision electricity meters by means of one single-phase reference instrument [2].

In circuit 2, the phase difference between the voltages and currents in every element of the tested instrument differ from those which exist in its normal connection (circuit 1). In circuit 3, the same phase differences are preserved within each element as in circuit 1. Therefore, it can be called an "artificial" Aron circuit. However, the phase differences between the voltages and currents in different elements differ from the corresponding phase differences in circuit 1. Hence, the torques due to the mutual effect of elements in an "artificial" Aron circuit differ from those existing in a normal Aron circuit.

It will be seen from Fig. 1b that in an Aron circuit connection the mutual-effect torque due to the interaction of the parallel and series circuits of the first and second elements, respectively, is equal to

\[ D_{12} = c_1 U_{12} I_3 \cos (U_{12} I_3) = c_1 U_{12} I_3 \cos (90° - \varphi). \]

The interaction torque of the series and parallel circuits of the first and second elements, respectively, is equal to

\[ D_{21} = c_2 U_{21} I_1 \cos (U_{21} I_1) = c_2 U_{21} I_1 \cos (90° + \varphi), \]

where \( c_1 \) and \( c_2 \) are the coefficients of proportionality.

Since the calibration and testing of wattmeters are made with a nominal voltage \( U_n \) and a symmetrical load with

\[ U_{12} = U_{23} = U_n; \quad I_1 = I_3 = I_n, \]

the resulting mutual effect torque is

\[ D_1 = D_{12} + D_{21} = U_n I (c_1 - c_2) \sin \varphi. \]  \hspace{1cm} (1a)

In a similar manner, the resulting mutual effect torque for circuit 2 is

\[ D_2 = U_n I (c_1 + c_2) \cos \varphi \]  \hspace{1cm} (1b)

and for the "artificial" Aron circuit, it is

\[ D_3 = U_n I \left[ -\frac{\sqrt{3}}{2} (c_1 - c_2) \cos \varphi + \frac{1}{2} (c_1 - c_2) \sin \varphi \right]. \]  \hspace{1cm} (1c)

Coefficients \( c_1 \) and \( c_2 \) depend on the position of the moving part in terms of rotation angle \( \alpha \).

The existence of mutual effect torques determines one of the components of the instrument's total error. The referred value of this component is equal to

\[ \gamma = \frac{D_n}{Tn}, \]

where \( D_n \) is the torque produced by the mutual effect of elements and is equal to \( D_1, D_2, \) or \( D_3 \), according to the circuit connections; \( Tn = kU_nI_n \) is the torque for a nominal power; \( I_n \) is the nominal current; \( k \) is a constant which does not depend on the rotation angle of the instrument's moving part.