HARMONIC ANALYSIS OF INFRASONIC ELECTRICAL SIGNALS
AND GRAPHIC CURVES
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In harmonic analysis of infrasonic electrical signals (from fractions to tens of Hertz), practical difficulties arise in producing selective elements with a high resolution.

In order to avoid this difficulty, the transformation of the tested signal spectrum is often produced by means of magnetic recording. The transformation of the spectrum is attained by raising considerably the speed of the magnetic tape in replay as compared with its recording speed.

The transformation of the spectrum to a higher frequency range makes it possible to use systems with a higher resolution and to reduce considerably the measuring time. This is due to the fact that for the same quality factor the transient processes entailed in retuning the analyzer to different frequencies are much shorter at higher frequencies.

If the duration of testing is not essential for the experimenter, it is possible to avoid spectral transformation and to use as a selective filter an ideal analyzer, by which we understand an analyzer suitable for producing a Fourier transformation.

It is known that the Fourier expansion coefficients are equal to:

\[ a_k = \frac{2}{T} \int_{0}^{T} f(t) \cos k \omega_0 t \, dt; \quad b_k = \frac{2}{T} \int_{0}^{T} f(t) \sin k \omega_0 t \, dt, \]

i.e., the amplitude and phase of the k-th harmonic is

\[ A_k = \sqrt{a_k^2 + b_k^2}; \quad \varphi_k = \arctg \frac{a_k}{b_k}, \]

or, inversely,

\[ a_k = A_k \sin \varphi_k, \quad b_k = A_k \cos \varphi_k. \]

If \( \varphi_k = 0 \), the coefficients of the \( b_k \) Fourier series are equal to the amplitude of corresponding harmonics in the spectrum of the tested signal. An ideal analyzer possesses in theory an unlimited resolution.

The Fourier transformation can be accomplished by means of a Hall converter placed in the air gap of the replay head [1].

It is known that the output voltage of a Hall converter is proportional both to the current which flows through it and to the strength of the field in which it is located. If the tested signal is recorded on an endless magnetic tape 2 (Fig. 1), the propulsion of this tape near the playback head 1 will change the induction in both air gaps of the head proportionately to the instantaneous value of the signal \( f(t) \). If converter 3 is fed from an external oscillator with a sinusoidal constant-amplitude current whose frequency can be smoothly tuned, thus providing at each required frequency \( \omega_k \) a phase shift \( \varphi_k = 0 \), the Hall converter output voltage \( E \) will be proportional to the product of the instantaneous values of the signal and the current, and the readings of an inertial pointer instrument will be proportional to the coefficients of the \( b_k \) Fourier series which are equal to the amplitude of corresponding harmonics \( A_k \).
Moreover,
\[ b_k = A_k \cos \varphi_k < A_k \quad \text{and} \quad b_{k \max} = A_k \text{ when } \varphi_k = 0. \]

Thus, by varying the phase at a given frequency, we can determine \( A_k \) for a maximum value of the output instrument reading (in practice, the phase can be set by a slow variation of the oscillator's frequency over a narrow range).

By changing the oscillator frequency we can make the output instrument record sequentially the direct voltage components which are proportional to the amplitude of the corresponding harmonics. From the frequencies read on the oscillator scale and the readings of the output instrument, it becomes possible to gauge the harmonic components of the output signal as well as their relative values.

It is obvious that the fundamental harmonic of the tested signal will equal the number of magnetic loop rotations per 1 sec. The knowledge of the fundamental frequency facilitates considerably the analysis, since all the oscillator frequencies which feed the converter must be multiples of the fundamental frequency.

The resolution and precision of measuring the amplitude of various harmonics in the above analyzer cannot, in practice, be made arbitrarily high.

This is due to an imperfect multiplication in the converter, an incomplete averaging in the measuring instrument and, finally, an imperfect shape of the sinusoidal current which flows through the converter.

The multiplication error is due to the fact that in working into an external load, the proportionality between the converter's output voltage and the magnetic induction is disrupted by changes in the internal resistance of the transducer produced by variations in the magnetic induction [2].

This deviation from linearity rises with a reduction for a given sample of \( \Delta \rho / \rho = f(B) \), where \( \rho \) is the initial resistivity of the converter, and \( \Delta \rho \) is the absolute variation of this resistivity in the magnetic field. A minimum deviation from linearity will correspond to an optimum value of the ratio \( r_l / r_i(B = 0) \), where \( r_l \) is the load resistance and \( r_i(B = 0) \) is the converter's internal resistance in the absence of a magnetic field.

Temperature has a considerable effect on the precision of multiplication, since it can change both the Hall constant and \( r_i \). In view of the short time required for harmonic analysis, temperature will virtually exert no influence on the precision of measurements. However, the current flowing through the converter should be selected on the basis of its permissible heating effect.

A satisfactory averaging will be produced by an instrument with a time constant \( \tau >> T \), where \( T \) is the revolution time of the magnetic tape. The speed of the tape propulsion can be chosen so as to meet this requirement.

With respect to the waveform of the current flowing through the converter, it is possible to obtain a nonlinear distortion factor of the order of 0.1-0.2%.

Among the random errors affecting precision in measuring harmonic amplitudes, the most important consists of inaccurate setting of the frequency and phase of the oscillator. Inaccurate setting of the frequency leads to very low-frequency oscillations of the output instrument pointer. Inaccurate setting of the phase produces an error in measuring the harmonic amplitudes, which is proportional to the term \( (1 - \cos \varphi) \), where \( \varphi \) is the error in setting the phase.

An important parameter of the analyzer consists of its sensitivity, characterized by the minimum harmonic amplitude which can be detected by the analyzer in the tested signal spectrum. This sensitivity is represented by the relationship

\[ \eta = \frac{E}{\Phi_f}, \]

where \( \Phi_f \) is the magnetic flux of the tape.