REMARKS ON THE ONE-PHOTON-PAIR ANNIHILATION IN STRONG MAGNETIC FIELDS *

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Abstract. We report specifically on a quantum electrodynamic feature of the one-photon-pair annihilation. Most of the calculation related with this process (not excluding other ones) have been carried out usually utilizing the wave functions obtained from the Dirac equation in the Landau gauge $\mathbf{A}_a = (0, Bx, 0)$, where $\mathbf{B}$ is oriented along the $z$-axis (Johnson and Lippman, 1949). Although, the eigenstates of the Dirac Hamiltonian as it was introduced by Johnson and Lippman, do not consider the coupling to the radiation field and consequently they reflect only for $p_z = 0$ the same linear combination of the two degenerate polarization states.

We report the transition rate function for the one-photon-pair annihilation in a strong magnetic field by using the Sokolov and Ternov eigenstate $|\pm\rangle$ as far as $p_z = 0$ and $N > 0$ is concerned. The difference between the expression for the transition rate by using the Sokolov and Ternov eigenstates and the one calculated by (Wunner et al., 1986), is just the function $I_{s,s'}$, which corresponds to the degeneracy of the orbit center of the electrons characterized by the quantum numbers $s$ and $s'$.

1. Introduction

Some transient $\gamma$-ray observations have revealed emission and absorption features in many $\gamma$-ray bursts. The most observed emission lines, seen in $\gamma$-ray bursts (Mazets et al., 1979; Teegarden and Cline, 1980; Mazets et al., 1981a,b; Klebesadel et al., 1982; Golenetskii et al., 1986; Atteia et al., 1987) fall in the energy range from 400 to 460 keV. Those lines could be gravitationally redshifted electron-positron annihilation produced near the surface of neutron stars. In particular there has recently been a renewed interest in the one-photon-pair annihilation in presence of a strong magnetic field which would help to understand the physics of the line formation in $e^- e^+$ annihilation in presence of strong magnetic fields. However, in this note we report specifically on a quantum electrodynamic feature of the one-photon-pair annihilation, which completes the calculation already done about this process. Most of the calculations related with this process have been usually carried out utilizing the wave functions obtained from the Dirac equation in the Landau gauge $\mathbf{A}_a = (0, Bx, 0)$, where $\mathbf{B}$ is oriented along the $z$-axis (Johnson and Lippman, 1949, hereafter referred as J.L.). Although, it should be mentioned that the eigenstates of the Dirac Hamiltonian, as it was introduced by J.L. do not consider the coupling to the radiation field and consequently they reflex only for $p_z = 0$ the same linear combination of the two degenerate polarization states. Furthermore, in some later publications where this eigenstates are employed they must be considered with reservation as far as cases with $p_z \neq 0$ and $N > 0$ are concerned. (Sokolov and Ternov, 1968) chose wave functions which are eigenfunctions that they called

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transverse polarization states of an operator \( \mu_z \) which may be interpreted as the component of the magnetic moment of the electron operator along the external magnetic field \( \mathbf{B} \). The gauge chosen in order to get such eigenfunctions is \( \mathbf{A}_b = (-B_y/2, B_x/2, 0) \), where \( \mathbf{A}_b = \mathbf{A}_a + \nabla \Lambda \), with \( \Lambda = -B_{xy}/2 \). The use of cylindrical polar coordinates seems useful for this particular problem. Furthermore, there are other sets of eigenstates used by Herold (1979) denominated helicity wave functions and Herold et al. (1983), denominated the proper polarization states of the electron, which take also into account the coupling of the electron to the radiation field by the interaction Hamiltonian, thus removing the degeneracy of the two polarization states in excited Landau levels. This all raises the question of how the form of the transition rate depends on the choice of the gauge for the external homogeneous constant magnetic field and how the corresponding spinors couple to the external field when they are applied to a radiation process. Moreover, Harding (1986) argues that the eigenstates used by Daugherty and Bussard (1980); which are the same as J.L., reproduce the correct results if the transition rate function is averaged over the electron and positron eigenstates.

2. The Transition Rate Function

In what follows, we present the transition rate function for the one-photon-pair annihilation in a strong magnetic field by using the Sokolov and Ternov eigenstate \( |\pm\rangle \) as far as \( p_z = 0 \) and \( N > 0 \) is concerned. The corresponding spinors are

\[
|\pm\rangle^N_{p_z} = \frac{e^{-i\epsilon \tau t} e^{i\epsilon k_3 z} e^{i(\ell - 1/2)\varphi}}{\sqrt{L_z} \sqrt{2\pi} 2\sqrt{2K_0 K}} \times
\begin{pmatrix}
\sqrt{K_0 + \zeta k_0} (\sqrt{K + \epsilon k_3} + \epsilon \zeta \sqrt{K - \epsilon k_3}) e^{-i\varphi/2} I_{N, S} \\
\zeta \sqrt{K - \zeta k_0} (\epsilon \zeta \sqrt{K - \epsilon k_3} - \sqrt{K + \epsilon k_3}) e^{i\varphi/2} I_{N-1, S} \\
\sqrt{K_0 + \zeta k_0} (\sqrt{K + \epsilon k_3} - \epsilon \zeta \sqrt{K - \epsilon k_3}) e^{-i\varphi/2} I_{N, S} \\
\zeta \sqrt{K - \zeta k_0} (\epsilon \zeta \sqrt{K - \epsilon k_3} + \sqrt{K + \epsilon k_3}) e^{i\varphi/2} I_{N-1, S}
\end{pmatrix}.
\]

(1)

Here the quantity \( \zeta = \pm 1 \) characterizes the two possible spin orientations of the electron relative to the direction of the magnetic field, \( K_0 = \sqrt{K^2 - k_3^2}, K = \sqrt{m^2 + 4\gamma N + k_3^2}, k_0 = m \) and \( \epsilon = \pm 1 \) for both possible signs of the energy, and \( I_{N, S}(\rho) \) are the normalized generalized Laguerre polynomials with argument \( \rho = \gamma r^2 \), see Ventura and Canuto (1977) for further properties of this function. Besides, \( \gamma = eB/2\hbar c \) and \( L_z \) is the periodic length along the \( z \)-axis. The S-matrix for this first order process reads

\[
S_{ij}^1 = (-ie) \int_{-\infty}^{\infty} dt \int d^3x (2V\omega)^{-1/2} U^*_\lambda(x, t) \alpha \cdot e^* U\lambda(x, t) e^{-ik \cdot x - \omega t}.
\]

(2)