A schematic for the Smith bridge, which is more correctly referred to as the Smith-Gautier circuit [1], is shown in Fig. 1a.

Such bridges are intended for measuring the resistance of four-terminal resistance thermometers with long connecting leads in the case of precision or reference temperature measurements.

It will be seen from the circuit in Fig. 1a that the Smith bridge consists of a modification of the normal "classical" Kelvin double-bridge. As compared with a normal Kelvin bridge the above circuit has two modifications. Firstly, the null-detector connecting points are interchanged with those of the supply source in order to provide more uniform loading and better matching of the null detector to the bridge output resistance, and to obtain as a result of this great sensitivity for the equipment as a whole. Secondly (and this is theoretically essential) the proportionality between the "external" and "internal" bridge arm ratios of $R_3 : R_2$ and $R_4 : R_4$, characteristic for a Kelvin bridge, is artificially unbalanced in the Smith bridge by a certain value, in order to compensate for the effect of the resistance of the measured resistance thermometer's wires.

The balance conditions of a "classical" Kelvin double bridge (Fig. 1b) can be expressed as [2, 3]:

$$R_x = R_0 \cdot \frac{R_3}{R_2} \left[ 1 + \frac{R_1}{R'_x + R_0} \left( \frac{R_1 - R_3}{R_1} - \frac{R_3 - R_4}{R_4} \right) \right],$$

where

$$R'_x = R_0 \cdot \frac{R_1}{R_2} \approx R_x.$$
expression (1) becomes equal to:

\[
\frac{R_1 - R_2}{R_1} = \frac{R_2 - R_4}{R_2},
\]

irrespective of the resistance of connection \(R_1\).

However, despite the fact that the actual basic parts \(R_1'\) and \(R_3'\) of bridge arms \(R_1\) and \(R_3\) can be adjusted with the precision required for any specific measurement, the resistance of connecting lead \(r_1\) in arm \(R_4\) can distort considerably the resistance of that arm (a similar, but less important, distortion in its final effect can occur in arm \(R_3\) due to the resistance of lead \(r_3\)). The value of \(R_1\) can also be increased owing to the resistance of lead \(r_4\). The latter circumstance raises substantially the requirement for the precision with which relationship (2) must be observed. This relationship can also be distorted if resistance \(r_1\) and \(r_3\) are not equal to each other with sufficient precision.

The basic idea of the Smith bridge consists of compensating the positive error in arm \(R_1\), due to the lead resistance \(r_1\), with the negative error in connection \(R_1\), due to the lead resistance \(r_4\), by artificially distorting (raising) the value of \(R_3\) and simultaneously reducing that of \(R_4\). This artificial distortion of the values of \(R_3\) and \(R_4\) is attained by transferring one of the bridge supply terminals from point \(C\) (see Fig. 1b), which corresponds to the Kelvin bridge circuit, to point \(B\) (see Fig. 1a), i.e., it is attained by adding a certain resistance \(R_{CB}\) (nominally equal to \(R_0\)) to arm \(R_3\) and subtracting the same value from arm \(R_4\).

In existing installations the Smith bridge parameters have the following values

\[
\begin{align*}
R_0 &= 10 \, \Omega \\
R_1' &= R_3' - R_{CB} = 1000 \div 10000 \, \Omega \\
R_2 &= R_4 + R_{CB} = 1000 \\
\end{align*}
\]

(for measuring \(R_x = 10 - 100 \, \Omega\)).

However, if the actual errors in the basic bridge arm components produce relationships of

\[
\begin{align*}
R_3' &= R_1' (1 + \alpha) + 10 \, \Omega \\
R_4 &= R_2' (1 + \beta) - 10 \, \Omega \\
\end{align*}
\]

then the final expression for the Smith bridge condition of balance will be

\[
R_x = R_0 \frac{R_1'}{R_2'} + 0.01 (r_1 - R_1) + \frac{0.99r_4 - r_3}{1000d} R_1 - R_0 \frac{R_5}{10d} (\alpha - \beta) R_1
\]

where \(d = 1 + 0.1R_0\); \(R_0\), \(R_1'\) and \(R_2'\) are the actual values of the corresponding bridge arm resistances, and \(R_1\) consists mainly of the connecting lead resistance \(r_4\).

If we assume for a similar measurement with a Kelvin bridge that

\[
\begin{align*}
R_3' &= R_1' (1 + \alpha) \\
R_4 &= R_2' (1 + \beta) \\
\end{align*}
\]

then the expression for a Kelvin bridge condition of balance will be

\[
R_x = R_0 \frac{R_1'}{R_2'} + 0.01r_1 + \frac{r_1 - r_3}{1000d} R_1 - R_0 \frac{R_5}{10d} (\alpha - \beta) R_1
\]

It can easily be seen by comparing expressions (4) and (6) that the last correction terms in the right-hand side of these expressions are exactly the same and the middle terms are virtually equal (the difference between 0.99 \(r_1\)