MEASUREMENT OF GRAVITATIONAL INTERACTION
PARAMETERS ON AN EARTH-ORBITING SATELLITE

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An analysis is presented of a space experiment whose purpose is to enhance the accuracy of measurement of
the gravitational constant \( G \), improve the estimates of the fifth force parameters \( \alpha \) and \( \lambda \), and finally to verify
the equivalence principle. The experiment involves exact measurements of the trajectories of a light body relative
to a heavy body on a drag-free Earth satellite. Estimations of the possible effect of various factors on the relative
motion trajectory are made. The equations of relative motion are derived for various types of satellite orbits.
Some preliminary estimates of the accuracy of the measurements are presented which show that the experiment
proposed seems to hold promise.

This article is the first of a number of papers devoted to the Satellite Energy Exchange (SEE) project — an experiment
on an artificial Earth satellite, the essence of which has been discussed in [1]. It is hoped that during the experiment the gravitational
constant will be measured with higher precision than has been achieved in laboratory experiments, and the inverse square law,
equivalence principle, and time-stability of the gravitational constant will be verified.

The current importance of these problems is due to a number of factors. First, of all fundamental physical constants
the gravitational constant has been measured with the least accuracy [2], this being a result of entanglement of various experimental
as well as theoretical problems. On the one hand, there is the smallness of the gravitational forces and their universality, as
a result of which it is impossible to shield the gravitational interaction; on the other hand, the theoretical status of the constant
is not clear since no unified theory of the fundamental physical interactions exists at present [3]. Moreover, some of the propositions
of the general theory of relativity are increasingly being subjected to critical analysis.

Second, in recent years doubt has been expressed regarding the classical basis of the theory of gravitation, viz., Newton’s
law. Essentially, this is due to the possible existence of a new macroscopic interaction with a range of the order of hundreds
of meters. It is assumed that the interaction may be of a gravitational nature and depend only on the mass of the interacting
bodies. In this case Newton’s inverse square law would be violated, and this could be interpreted as the gravitational constant
being dependent on the distance between the interacting bodies. Another possibility has also been suggested — a dependence
of the new interaction on the composition of the bodies. From this viewpoint an elucidation of the theoretical model of gravitational
interaction, even at the classical level, as well as any experimental data pertaining to the problem, would be of paramount importance
[4].

There are also a number of questions that arise in connection with the problem of time variation of the gravitational
constant. Usually implied are the cosmological variations related to the expansion of the universe [3, 5, 6]. The magnitude of
the variations depends on the Hubble constant and is quite small. Nevertheless, other time variations of the gravitational constant
may be possible and in particular those due to solar and other rhythms.

Finally, there is the problem of the equivalence principle, i.e., whether the free fall of all bodies is the same. On the
Earth the principle has been verified with a sufficiently high accuracy. However, the problem still persists. The cause of this
is that there may be a composition-dependent physical interaction.

Let us now consider the SEE project [1], according to which an artificial binary system consisting of a more massive
body, the "shepherd," and a lighter body, the "particle," would orbit the Earth. Thus, a three-body system consisting of the
Earth, "shepherd," and "particle" is being considered. In celestial mechanics many aspects of the three-body problem have been
discussed. We shall consider those that relate to the SEE project.
The gist of the SEE method is to make use of a very special class of orbits in the restricted circular three-body problem, viz., orbits of the horseshoe shape [7]. In this case the particle, moving along a somewhat lower orbit than the shepherd, overtakes it and then, as a result of the interaction, goes over to a higher orbit and begins to lag. Near the turning point the relative velocities of the particle and shepherd are small and thus such a binary system located in a drag-free capsule can be observed over a prolonged period of time. In contrast to the laboratory method, in the space method all external forces acting on the particle, which exceed considerably the force of interaction of the particle with the shepherd, can be eliminated. The tidal forces in the field of the Earth are of the same order as the particle—shepherd interaction. In the present and subsequent papers we analyze the new possibilities that the SEE concept opens up for determination of the gravitational interaction parameters. Another possibility of space determination of G is discussed in [8].

Lagrangian and Equations of Motion. In papers devoted to the three-body problem in celestial mechanics, the aim is usually to determine the motion of a body of infinitesimal mass m (satellite, asteroid) in the field of two massive bodies (e.g., the Sun and Jupiter) with comparable masses $M_i$ and $M_2$; this is the so-called restricted three-body problem. In the SEE project $M_1 = M_E$ ($M_E$ is the mass of the Earth), $M_2 = M = 500 \text{ kg}$ (heavy satellite) and $m = 100 \text{ g}$ (light satellite). Thus, $M/M_E \approx 10^{-23}$ and $m/M \approx 10^{-4}$. The effect of $M$ and $m$ on $M_E$ can certainly be neglected, whereas $M$ and $m$ are comparable. Thus the motion of the $(M, m)$ system in the static field of the Earth can be considered. As stated, the problem differs from the restricted three-body problem; however, a new small parameter $s/R$ appears (notations explained below). In previous studies the separations between the three bodies were assumed to be of the same order. In the SEE approach, however, the distance of bodies $M$ and $m$ from the center of the Earth is $\sim 10^7 \text{ m}$ while the distance between $M$ and $m$ is $s < 20 \text{ m}$.

With account for the perturbing potentials of external bodies (Sun, Moon, etc.) the Lagrangian of the "binary system" $(M, m)$ can be written as

$$L = \frac{1}{2} \left( MR^2 + m \tilde{r}^2 \right) + M\bar{V}(\tilde{r}) + m\bar{V}(\tilde{r}) + Mm\mathcal{U}(s),$$

(1)

where $\tilde{r}$ and $r$ are the geoconcentric radius vectors of masses $M$ and $m$ respectively; $\tilde{s} = \tilde{r} - \bar{r}$, $\bar{V}(\tilde{r})$ is the total potential of the Earth and other bodies of the solar system; $\mathcal{U}(s)$ is the interaction potential for bodies $M$ and $m$, and the potential does not necessarily have to be Newtonian. The quantity $s = |\tilde{s}| < 20 \text{ m}$ and the characteristic length for an appreciable change of the potential $V$ is of the order of the Earth’s radius $R_E$ since the leading Newtonian term $V_0$ in (1) is $V_0 = G M_E / R$.

The equations of motion for bodies $M$ and $m$ in an inertial reference system that follow from (1) are

$$\tilde{R}^i = V_i(\tilde{R}) - m \frac{\partial \mathcal{U}}{\partial s^i};$$

$$\tilde{r}^i = V_i(\tilde{r}) + M \frac{\partial \bar{V}}{\partial s^i}.$$

Their difference is the equation for $s^i(t)$, i.e., the equation of relative motion:

$$s^i = V_i(\tilde{R}) - V_i(\tilde{r}) + (M + m) \frac{\partial \mathcal{U}}{\partial s^i}.$$

(2)

Evidently, the relative motion is determined by the combination of tidal forces $[\bar{V}(\tilde{r}) - \bar{V}(\tilde{R})]$ and interaction forces related to the potential $\mathcal{U}$.

Some Estimates. We make some preliminary estimations of the various contributions to the right-hand side and the possible effect of the contributions on the trajectory of relative motion of the shepherd (mass $M$) and particle (mass $m$). Estimations of this type are required for an optical choice of the method of computer modeling of the trajectories and, in particular, for neglecting undoubtfully small contributions.

Separating out the dominant (spherical Newtonian) component in $\mathcal{U}$ and $\bar{V}$, we obtain

$$\bar{V}(\tilde{R}) = GM_E / R + \tilde{V}(R);$$

$$\mathcal{U}(s) = G / s + \tilde{U},$$

where $\tilde{V}$ includes the nonspherical part of the Earth’s potential and the potential of external bodies such as the Sun, Moon, planets, components of the satellite, and also the hypothetic non-Newtonian component of the geopotential; $\tilde{U}$ includes contributions due to the shepherd and particle not being ideally spherical after their preparation and also possibly a non-Newtonian component.

The explicit form of the total Newtonian potential of the Earth in the axial symmetry approximation is

$$\bar{V} = \frac{GM_E}{R} \left[ 1 + \sum_{n=2}^{\infty} J_n(R_E/R)^n P_n(\sin \psi) \right].$$

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