A focused paraxial beam of electrons with different energies in a device with a transverse control has the shape shown in Figure 1 with a greatly reduced scale along the x axis [1-3].

The following notations have been adopted in Figure 1: $a_0$ is the initial beam width; $CL$ is a cylindrical electron lens; $l$ is the distance from the lens to the anodes; $s_y$ is the direction of the deflected field; $l_1$ is the length of the deflecting system; $s$ is the distance between a deflecting system and the anodes; $a_{min}$ is the minimum width of a single electron beam with the smallest speed at the boundary of the anodes; $f_{min}$ is the focal distance of the lens for the electrons with the minimal speed; $A_1$ and $A_2$ are the anodes over which the beam current is distributed; $R$ is the output resistance.

The anodes $A_1$ and $A_2$ have been placed to the left of the focus of the slowest electron trajectories, since this location does not affect the amplitude characteristic's linearity requirement which consists of a unit (elementary) electron beam whose kinetic energies lie in a narrow band $\Delta e V_{min}$ being subjected to a minimal deviation, not exceeding at the anode line half a beam's width $a_{min}/2$. In placing the anodes beyond the focus $f_{min}$ this requirement is not met, since all the electrons of certain elementary beams which are focused at the boundary separating the anodes are transferred to one of the anodes by an insignificant voltage of the deflecting system. The proof of the amplitude characteristic's linearity requirement is based on the assertion that a change in each function (unit beam) proportional to the deflecting voltage produces a proportional change in their sum (complete beam). Deviations from proportionality of a single addend disrupts the proportionality of the entire sum.

Let us find the thickness $a_t$ of a unit beam at the anode boundary. By solving simultaneously the electron trajectory equation

$$\frac{x}{f_t} + \frac{2y}{a_0} = 1 \tag{1}$$

and the anode line equation $x = l$, we find after substituting $a_t/2$ for $y$, that

$$a_t = a_0 \left(1 - \frac{l}{f_t}\right) \tag{2}.$$

The focal distance of a single electron lens for a unit beam has the following form [4]:

$$f_t = \frac{V V_t + V_b}{\gamma} \tag{3}$$

where $V_T$ is the thermal potential of a moving electron; $V_b$ is the bias potential between the cathode and the lens; $\gamma = -\frac{4}{\int_{-\infty}^{+\infty} (V(x))^2 / V(x) dx}$ is the lens constant; and $V = V(x)$ is its potential function.

By substituting in (2) for $f_t$ its value from (3), we obtain

$$a_t = a_0 \left(1 - \frac{\gamma l}{V V_T + V_b}\right) \tag{4}.$$
The lens constant $\gamma$ can be determined from this expression if $V_T \to 0$ and $a_1 \to a_{\text{min}}$:

$$\gamma = \left(1 - \frac{a_{\text{min}}}{a_0}\right) \frac{V_V}{I}.$$

In reducing the accelerating voltage on the cathode it is necessary to take into account the Maxwellian energy distribution of the emitted electron. In this case the current of a unit electron beam whose kinetic energies lie in the range $\Delta e V_T$ is equal to

$$dJ_1 = I_0 \frac{e}{kT} \exp\left(-\frac{eV_T}{kT}\right) dV_T,$$

where $I_0$ is the total beam current; $e$ is the charge of an electron; $k$ is Boltzmann's constant; $T$ is the cathode temperature.

The current density produced by such a unit beam on the surface of one of the anodes is represented by the expression

$$d\sigma_1 = \frac{dI}{s_1} = \frac{I_0 \frac{e}{kT} \exp\left(-\frac{eV_T}{kT}\right) dV_T}{\frac{a_0b}{2} \left(1 - \frac{\gamma l}{\sqrt{V_T + V_b}}\right)},$$

where $s_1 = b \cdot a_1/2$ and $b$ is the beam width.

In a transverse electric field with the voltage $e_y$, each electron is subjected to the deflection $h_{y1}$, which depends on its energy $e(V_T + V_b)$ [4], with $h_{y1}$ being equal to

$$h_{y1} = \frac{l_1 e_y}{2 (V_T + V_b)} \left(\frac{l_1}{2} + s\right).$$

Here $l_1$ does not denote the length of the deflecting system's plates, but the length of a nominal capacitor without end effects whose value is equal to the capacitance of the deflecting system. Moreover, it should be noted that the error of the computation made below is determined by the error in finding the dimensions of such a nominal deflecting system.

The change in a single anode current due to a unit electron beam of the deflecting field is equal to

$$d(dJ_1) = d\sigma_1 d_s = d\sigma_1 b d_{h_{y1}} = \frac{l_1 \left(\frac{l_1}{2} + s\right) e_y I_0 \frac{e}{kT} \exp\left(-\frac{eV_T}{kT}\right) dV_T d\epsilon_y}{V_T + V_b - \gamma l \sqrt{V_T + V_b}}.$$

By substituting in the above equation for $\gamma$ its value from (5) we find after integration an expression for the deflecting current (departure from beam symmetry):

$$I_y = \frac{l_1}{a_0} \cdot \left(\frac{l_1}{2} + s\right) e_y I_0 \cdot \frac{e}{kT} \cdot \left(\frac{l_1}{2} + s\right) e_y \int_0^{\infty} \frac{\exp\left(-\frac{eV_T}{kT}\right) dV_T}{V_T + V_b - \left(1 - \frac{a_{\text{min}}}{a_0}\right) \sqrt{V_T + V_b}}.$$

The maximum deflection current $I_{y, \text{max}}$ can be found from this expression, provided that the amplitude characteristic is linear. This condition can be found from (5) and written in the form

$$\frac{a_{\text{min}}}{2} = \frac{l_1 e_y \left(\frac{l_1}{2} + s\right)}{2 (V_T + V_b)} \quad \left| \begin{array}{c} V_T \to 0 \\ \epsilon_y \to \epsilon_{y, \text{max}} \end{array} \right..$$