DESIGN FOR A HALL-TRANSDUCER MAGNETIC SYSTEM

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Considerable popularity has been recently acquired by transducers which use Hall emf generators for multiplying two quantities expressed in terms of currents or voltages. The transducer's output signal is directly proportional to the product of currents flowing through it and to the magnetic induction of the field in which it is located. Therefore, it is necessary for the above current and induction to coincide precisely in phase and in the shape of their curves with the input electrical quantities.

In order to obtain a high magnetic field strength, coils with steel cores and an air gap are normally used (the transducer's magnetic system). It is then rather difficult to ensure a linear relationship of the magnetic induction to the input current or voltage.

The nonlinear relationship of induction in the gap to the current flowing through the coil leads to changes in the conversion factor and is one of the causes of errors. Moreover, the magnetic system provides a phase shift in the gap induction with respect to the feeding electrical quantity. This shift is very large when the coil is supplied from a voltage source. If the coil is fed from a current source, it is normally possible to neglect this phase shift. In such cases only errors due to the nonlinearity of the magnetic characteristics of steel are examined in [1].

However, in studying and producing high-precision active power transducers it has been found that the error contributed by a magnetic system which is fed from a current source is due to a considerable extent to the phase shift produced by steel losses and to a smaller extent by the nonlinearity of the magnetic characteristic of steel.

Below we derive expressions which determine the phase shift and the nonlinear relationship, we analyze the errors due to the magnetic system which is fed from a current source, and we suggest a technique for designing magnetic systems of transducers.

The relationship of the induction phase shift δ in the air gap to the current in the coil can be represented as

\[ \tan \delta = \frac{P}{Q} \left( \frac{\mu'}{\eta} \sqrt{\left( \frac{P}{Q} \right)^2 + 1} \right) \]

where \( P \) are the steel losses in watts for a given induction; \( Q \) is the steel magnetization power, VA; \( \mu' \) is the relative permeability of steel; \( \eta \) is the ratio of the mean length \( l_s \) of magnetic flux lines in steel to the length \( l_a \) of the air gap.

For the case under consideration in which the core is assembled from punched strips without either butt joints or overlapping (with uniform induction throughout the cross section), we obtain

\[ \tan \delta = \frac{P}{Q} \left( \frac{\mu'}{\eta} \sqrt{\left( \frac{P}{Q} \right)^2 + 1} \right) = \frac{1}{1 + \frac{\mu'}{\eta} \sqrt{\left( \frac{P}{Q} \right)^2 + 1}} \cdot \frac{\tan \psi}{1 + \frac{\mu'}{\eta} \sqrt{\left( \frac{P}{Q} \right)^2 + 1}} \]

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The steel losses angle $\psi$ whose tangent is equal to the ratio of specific losses $p$ to the specific magnetizing power $q$ changes, like $\mu'$, according to the value of induction. Therefore, angle $\delta$ is variable and changes with the coil current within a certain range. It will be seen from the relationship of the loss angle to the field strength $\psi = f(aW_0/cm)$ that for electrical steel [2] the value of $\psi$ may attain $50^\circ$ or more.

Assuming that the induction in the air gap and in steel is the same, we find according to the total current law that

$$B = I \cdot \frac{W \mu_0}{l_0} \cdot \varepsilon,$$

where $W$ is the number of turns, $I$ is the current in the coil of the transducer's magnetic system, $\mu_0$ is the permeability of air.

The factor $\varepsilon = \mu'/(\mu' + \eta)$ represents the nonlinear relationship of induction to current in the coil of the transducer's magnetic system.

If a transducer's output signal is calibrated on the basis of its values corresponding to the nominal input currents $I_n$ and $I_{nh}$, respectively, of the coil and the Hall-transducer, then the maximum limiting error $\Delta U$ of the transducer will occur for $I_n = I_{nh}$:

$$\Delta U = \left| \frac{I}{I_n} \left[ \frac{\xi_n}{\xi} = \cos(\varphi - \delta) - \cos \varphi \right] \right| \cdot 100\% = \left| \frac{B}{B_n} \left[ \cos(\varphi - \delta) - \frac{\xi_n}{\xi} \cos \varphi \right] \right| \cdot 100\%.$$

Here $\xi$ and $\delta$ correspond to the induction for the current $I$, and $\xi_n$ for the current $I_n$; $\varphi$ is the phase difference between the input quantities of the transducer; $B_n = B_{mn}/\sqrt{2}$ is the given induction for the current $I_n$, which normally corresponds to the maximum value of the core permeability.

For very small values of $\delta$ it is possible to consider that $\delta = \sin \delta = \tan \delta$. Then, (4) will assume the form

$$\Delta U = \frac{B}{B_n} \left[ \cos \varphi \left( 1 - \frac{\xi_n}{\xi} \right) + \sin \varphi \tan \delta \right] \cdot 100\%.$$

The maximum of this function with respect to $\varphi$ is attained for $\tan \varphi = \tan \delta/(1 - \xi_n/\xi)$.

From the above we can obtain an expression for the relationship of the maximum error to the induction:

$$\Delta U = \frac{\Delta U''}{\cos \varphi} = \frac{B}{B_n} \sqrt{\left( 1 - \frac{\xi_n}{\xi} \right)^2 + \tan^2 \delta} \cdot 100\%,$$

where $\Delta U''$ is determined below in (9).

The maximum of this relationship represents the maximum error contributed by the magnetic system.

If we consider (for evaluating the error due only to the angle $\delta$) that factor $\varepsilon$ is constant, then the value of the above error can be represented as

$$\Delta U' = \frac{I}{I_n} \cdot \frac{2 \sin \frac{\delta}{2} \cdot \sin \left( \varphi - \frac{\delta}{2} \right)}{2 \sin \frac{\delta}{2} \cdot \sin \left( \varphi - \frac{\delta}{2} \right)} \cdot 100\% = \frac{B}{B_n} \cdot \frac{2 \sin \frac{\delta}{2} \cdot \sin \left( \varphi - \frac{\delta}{2} \right)}{2 \sin \frac{\delta}{2} \cdot \sin \left( \varphi - \frac{\delta}{2} \right)} \cdot 100\%.$$

Calculations have shown that a reduction in the ratio $I/I_n$ (or $B/B_n$) leads to an insignificant rise in $\sin (\delta/2)$. Therefore, in such instances the maximum effective error due to the phase shift in the induction with respect to the supply current will occur for $I = I_n$ ($B = B_n$), it will correspond to the angle $\varphi = (\pi - \delta)/2$ or $\varphi = (3\pi - \delta)/2$, and it will equal