One of the current problems in contemporary metrology is the development of methods and apparatus for the accurate measurement of physical quantities which change in time, i.e., the developments of dynamic measurements. At the input of a measuring system, considered as a passive four-terminal device, a process occurs with parameters or characteristics which are to be measured. Consequently, in contrast to static measurements in which it is necessary to measure some time-independent quantity and in which the experimenter, as a rule, does not disturb the character of the transient processes in the measuring system, in dynamic measurements we are concerned with measuring functions of time which must be reproduced to a specified degree of accuracy. As long as the response time of the measuring system plays a deciding role then it is understandable that dynamic measurements, particularly under conditions affected by external noise, have special characteristics not found in statistical measurements.

At the present time it is possible to single out two basic approaches to the problem of dynamic measurements.

The first of these two approaches is the apparatus correction of the input signal and essentially consists of attempting to obtain a signal with possibly less distortion than the input signal at the measuring device output. This is done by including various correction elements or components in the system. At the same time it is necessary to take into account the possibility of physically realizing the required correction.

The other approach to metrology and dynamic measuring apparatus presents greater possibilities by broadening the class of functions which can be measured. Actually, if the output device of the measuring system maintains the necessary predominance of signal over noise, then the problem of the measurement of practically any physically realizable unknown function \( x(t) \) reduces to the measurement of the output function \( y(t) \) and to the calculation of \( x(t) \) from \( y(t) \) and from the known properties of the system. The latter are determined by a different method. For example, a linear apparatus can be defined either by a system of the usual differential equations with constant coefficients, or by its pulse characteristic or by its complex transmission coefficient, etc. All these properties are uniquely related to each other and are equivalent. This relationship is expressed with the help of a Fourier transformation. Thus, a direct Fourier transformation of the pulse characteristic leads to the complex transmission coefficient which, in turn, can be obtained from the differential equations for the apparatus [1].

In this paper, the pulse characteristic \( h(t) \) of the apparatus is used, i.e., the response of the apparatus to a delta-function input. The pulse characteristic is related to the input and output functions by the following equation:

\[
y(t) = \int_{t_0}^{t} h(t - \tau) x(\tau) d\tau, \quad (1)
\]

where \( x(t) = 0 \) for \( t < t_0 \).

Consequently, if \( h(t) \) is known, then the unknown function \( x(t) \) can be found from Eq. (1) for \( y(t) \). In the general case, the solution of integral Eq. (1) does not satisfy the proper requirements, i.e., existence, uniqueness, and stability. However, by applying several reasonable restrictions to \( x(t) \) and \( h(t) \), e.g., finiteness in time and magnitude, the problem becomes uniquely determined [2].

The simplest methods of solution, which use a transformation to a system of linear algebraic equations for values of \( y(t) \) chosen at the points \( t_j (j = 1, 2, \ldots, n) \) such that \( x(t) \) is defined by some \( m \) numbers, can be seen to be practically ineffective for the case \( n = m \) because the response of \( y(t) \) is usually not known exactly due to different kinds of interference and, in view of this fact, the output signal has the form \( y^*(t) = y(t) + \xi(t) \), where \( \xi(t) \) is noise. A further increase in the accuracy of measurement is possible by a statistical analysis. In

particular, the number of readings \( y^* (t_j) \) is chosen to be much greater than the number \( m \) and the resulting mutually exclusive system of equations is solved by the method of least squares. The problem of the best treatment of the response \( y^* (t) \) is the same as the problem of optimum filtering and can be formulated as a variational problem to determine the functions \( g_i (t) \) such that

\[
a_i = \frac{1}{T} \int_{t_0}^{T} y^* (t) g_i (t) \, dt
\]

and

\[
x^* (t) = \sum_{i} a_i q_i (t)
\]

approximates \( x(t) \) according to some criterion. The most convenient criterion for an orthogonal series is minimizing the value of the root-mean-square error

\[
\varepsilon = \left( \frac{1}{T} \int_{t_0}^{T} [x(t) - x^*(t)]^2 \, dt \right)^{1/2}.
\]

When the correlation function of the noise \( \xi (t) \) is known, the solution of this problem is not complicated.

Above, we considered the pulse characteristic of a certain apparatus. We will consider the possibility of its detailed determination because the accuracy of measuring \( x(t) \) depends critically on the accuracy with which \( h(t) \) is determined. Consequently, the determination of \( h(t) \) to a specified accuracy will solve the problem of accuracy in the dynamic measurement of physical quantities. Of course in the construction of an apparatus its preliminary properties are known, and the expected pulse characteristic can be approximately calculated. However, exact measurements require the exact determination of \( h(t) \) from experiment. It is evident that Eq. (1) can be used for this purpose if the experimenter will either use some reference signal \( x_0 (t) \) with accurately known characteristics so that \( h(t) \) can be determined from \( x_0 (t) \) and \( y(t) \), or use an apparatus with a precisely known characteristic \( h_0 (t) \) so that the problem reduces to the solution of a system of integral equations in the unknowns \( x(t) \) and \( h(t) \):

\[
\begin{align*}
y(t) &= \int_{t_0}^{t} [h(t - \tau) x(\tau) \, d\tau, \\
y_0 (t) &= \int_{t_0}^{t} h_0 (t - \tau) x(\tau) \, d\tau.
\end{align*}
\]

It should be noted that the existence of an apparatus with an accurately known characteristic \( h_0 (t) \) would remove the problem of accurately determining \( h(t) \) by the method of its measurement. Therefore, the problem of the accurate determination of \( h(t) \) has a meaning only when a standard apparatus is not present and has a wide generality because one arrives at this very problem from the necessity of constructing an apparatus with the reference characteristic \( h_0 (t) \).

This problem is most successfully solved by the use of a dynamic reference signal, i.e., a function of time with precisely known parameters and calibrated by a fixed reference or by a simulation of the same physical quantity without the use of the usual intervening devices for which a knowledge of dynamic characteristics or the application of correction circuits are required.

It should be possible to propose the following method to obtain a reference signal \( x_0 (t) \) which represents some time-dependent physical quantity (pressure, force, temperature, etc.). Let us assume that an apparatus has been built which has the required dependence of some physical quantity of interest on the spatial coordinate \( l \), i.e., the function \( x_0 (l) \). This function is independent of time and can be precisely measured by fixed instruments at any point \( l \) along some spatial trajectory.

Moving along the chosen apparatus trajectory for which the pulse characteristic is to be determined and knowing the change of velocity \( v(t) \) in time, it is not difficult to determine the function

\[
x_0 (t) = x'_0 (l) = x'_0 \left[ \int_{l}^{t} v(\tau) \, d\tau \right].
\]