HYDRAULIC FORCES IN PISTON SYSTEMS OF INSTRUMENTS
USING PISTONS WITHOUT GASKETS

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The first attempts to calculate the pressing-out forces and those of liquid friction in a reference instrument with its piston rotating with respect to its cylinder \([1, 2]\) were based on the direct use of the fundamental mechanism of the hydrodynamic lubrication theory which relates the pressing-out forces and those of liquid friction with the eccentricity of the piston with respect to the cylinder. Moreover, it was assumed that the axes of the piston and the cylinder always remain parallel to each other. Despite the similarity between a piston and a cylinder, and a journal bearing, the peculiarities of the reference piston instrument's operation provide special conditions for the formation of pressing-out forces.

The theoretical difference between the operation of piston instruments and that of journal bearings consists in the fact that in the former the main load is directed along the axis of the piston, and in the latter perpendicular to the axis of rotation. However, under actual operating conditions instruments using simple pistons without any packing are also loaded by forces perpendicular to the piston axis. These forces arise owing to the fact that it is impossible to attain an ideally accurate adjustment and centering of a piston and a cylinder, of weights, etc.

![Fig. 1.](image)

Moreover, journal bearings, as a rule, are placed symmetrically with respect to the effective load, whereas in piston instruments such a location of the load is, in the majority of cases, difficult to attain. It is therefore necessary to examine the case when the piston is loaded both by a force and a moment.

**Hydrodynamic pressing-out forces existing with skewed piston and cylinder axes.** In the case when the load acting on the piston consists of a force and a moment, it is necessary in order to preserve balance for a hydrodynamic pressure force to act on the piston from the side of the lubricating layer, thus balancing the external load, which is only possible when the piston and cylinder axes are skewed. A rigorous solution of this problem presents considerable difficulties. It is advisable to use an approximate solution with subsequent experimental investigation, since it is impossible to account for all the factors existing in the gaps of actual instruments.

If one assumes that Sommerfeld's formula holds for an elementary ring \(dz\) of the piston, the value of the hydrodynamic pressing-out force acting on that ring is determined by the formula

\[
dP = 12\pi \frac{r^3}{h^2} \frac{\alpha}{(2 + \alpha^2) \sqrt{1 - \alpha^2}} dz,
\]

where \(r\) is the piston radius; \(h\) is the radial clearance between the piston and the cylinder; \(\omega\) is the angular velocity.
of the piston rotation with respect to the cylinder; \( \mu \) is the dynamic viscosity of the pressure liquid; \( \alpha = e/h \) is the relative eccentricity of the piston in the area of the elementary ring.

The solution of the problem is then reduced to the summing of these elementary forces along the axis of the piston, with the boundary conditions determined by the nature of the load. The assumption thus made approaches closely actual conditions, since in view of the smallness of the clearance it is possible to consider that within the range of the elementary ring the piston and cylinder axes are parallel. This method of an approximate solution is widely used in the general theory of instruments using pistons without packing [3].

For further simplification of the problem let us neglect the effect of the liquid escaping through the ends of the clearance, the deformations of the clearance and the changes in the viscosity of the liquid under pressure.

If the cylinder is loaded with an external force \( F \) and a moment \( M \), the balance equations of the cylinder will become

\[
\int dP = F,
\]

\[
\int dP(z-\Delta) = M,
\]

where \( \Delta \) is the distance from the origin of the coordinates to the point of intersection between the piston and the cylinder axes (the origin of coordinates lies at the center of the sliding pair).

The integration of the balance equations provides the expression

\[
F = F_0\psi, \\
M = F_0\varphi, 
\]

where \( I \) is the length of the clearance between the piston and the cylinder;

\[
F_0 = 12\pi \mu \frac{r^2}{h^2} \omega l; \\
\psi = \frac{1}{V(\alpha_1-\alpha_2)} \left( Ar\sqrt{\frac{1-\alpha_2^2}{3}} - Ar\sqrt{\frac{1-\alpha_1^2}{3}} \right); \\
\varphi = \frac{1}{(\alpha_1-\alpha_2)^2} \left[ \arcsin\alpha_1 - \right. \\
- \sqrt{\frac{2}{3}} \arctg \left( \sqrt{\frac{3}{2}} \sqrt{\frac{\alpha_1}{1-\alpha_1^2}} - \right. \\
+ \sqrt{\frac{2}{3}} \arctg \left( \sqrt{\frac{3}{2}} \sqrt{\frac{\alpha_2}{1-\alpha_2^2}} \right) \right]; \\
\alpha_1 \text{ and } \alpha_2 \text{ are the relative eccentricities of the piston axis at the upper and lower end planes of the cylinder.}
\]

It follows from the balance equations that \( \zeta = \varphi/\psi = M/FI \), i.e., the relationship between \( \alpha_1 \) and \( \alpha_2 \) is only determined by the nature of the external load \( \alpha_2 = f(\alpha_1, \zeta) \). Two extreme cases are possible.

1. The cylinder is loaded by force \( F \). There is no moment of external forces (\( \zeta = 0 \)). In this case the load coefficients assume the form \( \varphi = \varphi(2 + \alpha^2)\sqrt{1 - \alpha^2} \), \( \psi = 0 \), i.e., the solution is reduced to Sommerfeld's formula.

2. The piston is loaded by a pure moment \( M \) (\( \zeta = \infty \)). The load coefficients are equal to \( \psi = 0 \),

\[
\varphi = \frac{1}{2\alpha_1^2} \left[ \arcsin\alpha_1 - \sqrt{\frac{2}{3}} \arctg \left( \sqrt{\frac{3}{2}} \sqrt{\frac{\alpha_1}{1-\alpha_1^2}} \right) \right].
\]