It was found that it is very fruitful and effective to use information-theory methods for investigating measuring systems, since measurement is a process of receiving and transforming information about the measured quantity. One of the most important conclusions of the information theory consists of the fact [1] that measurement errors are inevitable, since they represent one of the aspects of the actual physical process of measurement. Thus, they are inevitable theoretically and, therefore, must be the subject of profound studies.

According to the information theory described in [2], it is possible to consider that the only interference which affects the transmission of information consists of a random measurement error statistically unrelated to the measured quantity. The information content $\Delta I$ lost in measurements and expressed in a logarithmic form can be determined as the entropy of error $\Delta X$:

$$\Delta I = H(\Delta X) = - \int_{-\infty}^{\infty} p(x) \ln p(x) \, dx,$$

where $p(x)$ is the probability density of the error distribution as a random quantity.

For a uniform error distribution in the range $(-\Delta, \Delta)$ the entropy is equal to

$$H(\Delta) = - \int_{-\Delta}^{\Delta} \frac{1}{2\Delta} \ln \frac{1}{2\Delta} \, dx = \ln 2\Delta,$$

i.e., to the logarithm of the bandwidth within which the error exists.

The simplicity of (2) led P. V. Novitskii [2-5] to adopting the concept of an entropy error based on replacing the initial error with an arbitrary distribution law by a certain equivalent error with a uniform distribution and the same entropy:

$$H(\Delta X) = - \int_{-\infty}^{\infty} p(x) \ln p(x) \, dx = H(\Delta) = \ln 2\Delta,$$

whence

$$\Delta = \frac{1}{2} e^{H(\Delta X)}.$$

Therefore, the entropy error is defined as half the bandwidth of an equientropy (i.e., an equivalent entropy) error with a uniform distribution.

If the distribution of a continuous measured quantity in the range $L$ is assumed to be uniform, the initial entropy will be equal to

$$H_0 = - \int_{0}^{L} \frac{1}{L} \ln \frac{1}{L} \, dx = \ln L,$$

and the residual entropy, according to (2) and (3), will be

$$H_1 = H(\Delta X) = \ln 2\Delta.$$
From the above it is possible to obtain one of the mathematical expressions for the fundamental essence of measurements, which consists of reducing (narrowing) the band of indeterminacy in the values of the measured quantity from dimension \( L \) to \( 2\Delta \). The information content received in measurement is equal to the difference between the initial and residual entropies:

\[
I = H_0 - H_1 = \ln \frac{L}{2\Delta} = \ln N,
\]

where \( N = L/2\Delta \) is the number of various gradations of the measured quantity.

Thus, we arrive at the concept of information precision \([2]\) whose quantitative expression \( A \), for an assumed uniform distribution of the measured quantity, coincides with number \( N \) of distinguishable gradations:

\[
A = N = \frac{L}{2\Delta} = \frac{1}{2\gamma},
\]

where \( \gamma = \Delta/L \) is the relative entropy error of measurement.

The entropy error is related to the entropy power \( P \) of the error considered as an interference by the following expression:

\[
\Delta = \sqrt{\frac{\pi e P}{2}}.
\]

It is obvious that in theoretical computations of the information content obtained in measurement it is always necessary to use entropy errors determined from (4). Moreover, the problems of adding several entropy errors have as yet to be worked out and investigated. The entropy error is also an objective and most effective criterion for comparing information characteristics of measuring systems and instruments which have random errors with different distribution laws.

The information theory is suitable for developing theoretically new methods of normalizing precision and evaluating the metrological characteristics of measurements and measuring equipment by using entropy instead of probability criteria. In fact, the existing generally-accepted methods for evaluating the precision of measuring equipment by its limiting confidence error cannot be considered strictly substantiated, since the selection of the fiducial probability level which must serve as a measure of the obtained measurements' trustworthiness is conditional and is attained without evaluating the content of the obtained information. Thus, criterion \( 3\sigma \) for a normal error distribution law is based on an intuitive assumption that the probability of random errors appearing in excess of \( 3\sigma \) is virtually zero. In this connection it seems better to normalize the precision of measurements and measuring instruments by their basic characteristic, the quality of information received in measurements.

However, the reduction of the true error to an equivalent error form (for instance with a uniform or normal distribution) obviously cannot change the initial probability distribution law and the entropy distribution law determined by it. Therefore, side by side with the entropy error it is also advisable to adopt precision criteria based on the information content obtained in measurements and at the same time take into consideration the particular features of the error distribution law.

In the majority of cases the problem of normalizing the measuring equipment precision consists of determining in any given manner the range which under actual operating conditions should virtually not be exceeded by the equipment error. In normalizing the precision of a measuring instrument by its maximum error \( \delta \), the random errors outside the band \((-\delta, \delta)\) are neglected. Therefore, in using the instrument, the errors whose probability of appearing amounts to

\[
\Delta P = 1 - \int_{-\delta}^{\delta} \rho(x) \, dx
\]

are not taken into consideration; neither is the error entropy part which is represented by the expression

\[
\Delta H = - \int_{-\infty}^{\infty} \rho(x) \ln \rho(x) \, dx - \int_{-\delta}^{\delta} \rho(x) \ln \rho(x) \, dx.
\]