In designing springs (elastic bodies) for elastic dynamometers it is necessary to meet two contradictory requirements. On the one hand, in order to improve the metrological properties of dynamometers, it is desirable to have a large measurable deformation of the spring, and on the other hand its dimensions and weight should be minimal.

In dynamometers with a nominal force \( P \) and a given (desirable) measured deformation \( \delta_s \) of the spring, the dimensions and weight of this spring can vary within adequately large limits. This is due to the mechanical properties of the spring materials and, above all, to the elastic limit (of proportionality) \( \sigma_c \), as well as to the shape and design of the spring.

It is obvious that, for the remaining conditions being equal, the weight of the spring can be reduced for a rising elastic limit \( \sigma_c \). At the same time instances are known when dynamometers made from the same materials in different shapes have very different weights. Bearing this in mind the problem of determining and selecting the shape of the spring deserves serious consideration. This is especially important in designing springs for tens of millions of newtons.

In order to determine the advisability of a given spring shape it is necessary to have a comparison specimen. Such specimens may consist of an elastic body subjected over its entire volume to a stress for which the stored potential energy of deformation has a maximum value. Considering that for the three stressed conditions of 1) \( \sigma_1 = \sigma \) and \( \sigma_2 = \sigma_3 = 0 \); 2) \( \sigma_1 = \sigma_2 = \sigma_3 = \sigma_0 \); and 3) \( \sigma_1 = \sigma \) and \( \sigma_2 = \sigma_3 = 0 \) the ratio of the potential energies is constant,* the specimen can be made of a spring in the shape of a prismatic bar subjected to pure tension or compression (see Fig. 1a):

\[
\sigma_1 = \sigma \quad \text{and} \quad \sigma_2 = \sigma_3 = 0.
\]

The spring shape can be determined by comparing its mass with that of the specimen, provided that the following conditions are met.

1. The measured deformation \( \Delta l \) of the specimen (see Fig. 1a) should be equal to that of the spring

\[
\Delta l = \delta_s,
\]

but it should then be borne in mind that, as distinct from the specimen's condition the measured deformation \( \delta_s \) in a general case does not coincide with the direction of applied force \( P \) (in Fig. 1b this direction is a-b). Then:

\[
\delta_s = C \Delta l_s,
\]

where \( C \) is a coefficient depending on the design of the spring, \( \Delta l_s \) is the deformation of the spring in the direction of force \( P \).

2. The maximum permissible stress \( [\sigma] \) of the spring should be equal to that of the specimen. Let us compare the masses by taking the ratio of the spring deformation per unit of its mass to the specimen deformation per unit of its mass [2]:

\[
m = \frac{\Delta l_s}{\rho V_s}, \quad \frac{\rho V_s}{\rho V_0}
\]

*By substituting in the general expression for potential energy [1] the value \( \mu = 0.3 \), we find for the above three cases that \( u_1 : u_2 : u_3 = 3.6:1.2:1 \).
where \( \rho \) is the density of the spring and specimen material, \( V_s \) and \( V_0 \) are the volumes of the spring and specimen respectively.

Let us call ratio \( m \) the form factor. The weight of the spring drops with a rising \( m \) for the same strength and precision.

By multiplying and dividing the right-hand side of (3) by \( P/2 \) we obtain

\[
m = C \frac{P \Delta l_s}{2V_s} = C \frac{\Delta l}{u_0} = \frac{V_s}{V_0},
\]

where

\[
u_0 = \frac{P \Delta l}{2V_0} = \frac{[\sigma]_0}{2E}
\]

is the specific potential energy of the specimen;

\[
u_s = \frac{P \Delta l_s}{2V_s} = K \frac{[\sigma]_s}{2E}
\]

is the specific potential energy of the spring.

Design coefficient \( K \) in (6) depends on the shape of the spring and the type of its stressed condition. In view of the fact that for the second type of stressed condition \([\sigma]_s = [\sigma]_0 \) by definition, expression (4) can be rewritten

\[
m = CK.
\]

Let us calculate \( m \) for an Olsen ring dynamometer with a nominal force \( P \approx 0.0 \) kN. Let us denote with \( E \) the elasticity modulus, with \( r \) the radius of the mean line of the ring, with \( I = bh^3/12 \) the moment of inertia of the ring rectangular cross section, and with \([\sigma]_0 = [\sigma]_k \) the maximum permissible tension in the spring.

The deformation of the annular spring is measured in the direction of the effort and, therefore,

\[
C = \frac{\Delta l_s}{\Delta l} = 1.
\]

The value of the sag \([\delta] \) is

\[
\Delta l_s = \left( \frac{\pi^2 - 8}{4\pi} \right) \frac{Pr^3}{EI}.
\]

The potential energy accumulated in the ring during deformation is

\[
U_s = \frac{P \Delta l_s}{2} = \frac{(\pi^2 - 8)}{4\pi} \frac{Pr^3}{2EI}.
\]

The maximum bending moment at the point where the force is applied \([\delta] \) is

\[
M_{\text{max}} = \frac{Pr}{\pi} = [\sigma] W = [\sigma] \frac{bh^3}{6}.
\]

By determining from the above the product \( Pr \) and raising it to the second power we obtain

\[
Pr^3 = \frac{[\sigma]^2 h^3 \pi^2}{36}.
\]

By substituting in (9) for \( Pr \) its value obtained from (11) we find that

\[
U_s = \frac{\pi^2 - 8}{24} 2\pi rh^3 \frac{[\sigma]^3}{2E} = KV_s \frac{[\sigma]^2}{2E},
\]

where

\[
k = \frac{\pi^2 - 8}{24} \approx 0.977.
\]