Analog-digital conversion is accompanied, contrary to analog conversion, by its inherent level-quantization error. In many instances when measuring systems containing both analog-digital converters (ADC) and analog converters (AC) are being analyzed, it is very convenient to represent AD converters in the form of A converters whose input is fed by a certain supplementary signal consisting of quantization noise which is equivalent in its effect to level quantization. In a general case the statistical characteristics of this signal depend on the instrumental noise of AD converters and the measured signal and, in this sense, the level-quantization error becomes a compound AD converter's characteristic (intrinsic and instrumental). In the present article we determine the ADC quantization error characteristics and make a comparative analysis of the quantization error of AD converters of the synchronized (ADCs) and unsynchronized (ADCu) types [1] under a static operating condition (with an unchanging input quantity), with the object of a rational demarcation of their application spheres. An analysis is also made of the methods for evaluating the total conversion error of ADs and ADu converters subjected to an aggregate of random variable factors which affect the conversion error.

Level-Quantization Error. On the basis of representing an AD as an A converter whose input is fed with a quantization noise, it is possible to consider the total absolute random conversion error \( \delta_5 \) referred to the ADC input as consisting of absolute random error \( \delta \) due only to ADC instrumental noise and the effect of external variable random factors and to quantization (noise) error \( \delta_5 \):

\[
\delta_5 = \delta + \delta_5 \quad (1)
\]

In analyzing the quantization error it is convenient to combine \( \delta \) with the measured quantity \( Y_0 \) and to consider that the input of an ideal AD converter is fed with a certain equivalent signal \( Y_5 \) equal to the measured signal \( Y_0 \) and error \( \delta \):

\[
Y_5 = Y_0 + \delta \quad (2)
\]

The value of the measured quantity can be represented in the form:

\[
Y_0 = (n_0 + \Delta n_0) \mu,
\]

where \( \mu \) is a quantum, a unit of the lowest ADC discrete output signal (code) order; \( n_0 \) is a whole number of quanta contained in \( Y_0 \); and \( 0 < \Delta n_0 < 1 \).

Let us denote the converter's discrete output signal by \( y_k = k \mu \), where \( k = 0, 1, 2, \ldots, k_{\text{max}} \).

The level-quantization error \( \delta_5 \) is

\[
\delta_5 = y_k - Y_5 \quad (3)
\]

From (1) and (2) we obtain

\[
\delta_5 = y_k - Y_0 \quad (4)
\]

Let us evaluate the statistical characteristics of component \( \delta_5 \) of the total error \( \delta_5 \) for an ADs and an ADu converter.

The problem of level quantization has been discussed in [2-6]; however, the known results cannot be used directly. In some of the works the basic results which characterize the quantization error by means of univariate or bivariate distribution laws (correlation functions, variances and mean values) hold only if input signal variance \( \sigma^2 \) is large as compared with the lowest order ADC code unit squared, \( \mu^2 \); whereas in other works the errors of ADu converters which are widely used in practice are not dealt with. The limiting condition \( \sigma \gg \mu \) adopted in a number of works is justifiable in the case when the measured quantity is represented in the form of a random process at the ADC input and is characterized by variance \( \sigma^2 \). In the relevant analysis of the quantization error, with the

instrumental noise taken into consideration, there are no reasons whatsoever for considering in static measurements that \( \sigma \gg \mu \), but it is necessary to take into account that the root-mean-square deviation \( \sigma \) of error \( \delta \) is virtually commensurate with or smaller than a unit of the ADC smallest order code.

Let us examine the most general quantization error characteristic in static independent measurements, which consists of the probability density distribution. Let us evaluate this function without applying limitations on the relationship between \( \sigma \) and \( \mu \) and omitting the edge effect.

The probability density distribution of the quantization error can be evaluated on the basis of (3) from the known probability density distribution of signal \( Y_\delta \) and the conditional probability of the output code, assuming the value of \( y_k \) if \( Y_\delta \) assumes a certain definite value. According to (3), the probability \( p'_{\delta,Y_\delta} \) that the value of the quantization error lies within the infinitely small range of \( \delta'_{Y_\delta} - \varepsilon \) to \( \delta'_{Y_\delta} + \varepsilon \), if signal \( Y_\delta \) lies with probability \( p'_{Y_\delta} \) within the infinitely small range of \( Y'_\delta - \varepsilon \) to \( Y'_\delta + \varepsilon \) and the ADC output signal is \( y_k \), is equal to

\[
p'_{\delta,Y_\delta} = \sum_{k=0}^{k_{\text{max}}} p'_{Y_\delta} p(\frac{y_k}{Y_\delta}),
\]

where \( p(y_k / Y_\delta) \) is the conditional probability that the ADC output signal will assume value \( y_k \) if the input signal assumes the value of \( Y_\delta \).

The summation with respect to \( k \) in (5) was made according to the theory of summation (events consisting in the appearance of readings \( y_k \) are incompatible [2]).

From the principle of the ADCs operation it follows that,

\[
p(\frac{y_k}{Y_\delta}) = \begin{cases} 1 & \text{for } y_k \text{ which complies with inequality:} \\ & Y_\delta - \mu < y_k < Y_\delta; \\ 0 & \text{for all remaining values of } y_k. \end{cases}
\]

Let us denote by \( f_1(\delta) \) the probability density distribution of error \( \delta \) which is referred to the input. Taking into consideration that \( Y_\delta = Y_0 + \delta \), henceforth we shall bear in mind that

\[
I_1(\delta) = I_1(Y_\delta).
\]

By expressing (5) in terms of densities, and taking into consideration that

\[
p'_{\delta,Y_\delta} = f(\delta,Y_\delta) \Delta Y_\delta; \quad p'_{Y_\delta} = I_1(Y_\delta) \Delta Y_\delta,
\]

we obtain

\[
f(\delta,Y_\delta) = \sum_{k=0}^{k_{\text{max}}} f(\delta,Y_\delta)p(\frac{y_k}{Y_\delta}),
\]

or, accounting for (6),

\[
f(\delta,Y_\delta) = \sum_{k=0}^{k_{\text{max}}} f(\delta,Y_\delta) < y_k < y_k + \mu(Y_\delta).
\]

By using (1), (2) and (7), let us rewrite (8) in the form

\[
f(\delta,Y_\delta) = \sum_{k=0}^{k_{\text{max}}} I_1(y_k - Y_0 - \delta,Y_\delta) \text{ for } -1 < \delta < 0.
\]

Quantization error \( \delta_\delta \) results from the quantization of signal \( Y_\delta \) and is functionally related to it. Formula (9) expresses this relationship. Thus, the law of the quantization error distribution in a general case depends on the distribution law of signals \( Y_\delta \). It can be shown that this relationship is most pronounced for small variance \( \sigma^2 \) of signal \( Y_\delta \), whereas for condition \( \sigma \gg \mu \) the distribution law of the quantization error tends towards a uniform distribution within the quanta irrespective of the distribution law of signal \( Y_\delta \).