ON THE FLOW OF AN ELECTRICALLY CONDUCTING FLUID
AND HEAT TRANSFER ALONG A PLANE WALL
WITH PERIODIC SUCTION

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ABSTRACT. The purpose of this paper is to consider the behaviour of a three-dimensional laminar boundary layer flow of an incompressible, electrically conducting Newtonian fluid past a plane wall in the presence of a transverse suction velocity distribution applied at the wall. The components of the wall shear stress and heat transfer, using the method of perturbation, are obtained. The variations of these quantities with the Prandtl number and a magnetic parameter are also investigated.

SOMMARIO. Scopo di questo lavoro è di studiare il comportamento di un flusso di stato-limite laminare tridimensionale e di un flusso newtoniano incompressibile elettroconduttore lungo una parete piana in presenza di una distribuzione di velocità trasversale di aspirazione applicata alla parete.

Si determinano le componenti di tensione tangenziale sulla parete e la trasmissione di calore usando un metodo perturbativo. Si studiano anche le variazioni di queste quantità con il numero di Prandtl e con il parametrop magneticco.

KEY WORDS. Magnetohydrodynamic, Heat transfer, Periodic suction.

1. INTRODUCTION

It is well known now that turbulence in boundary layers increases the drag. One of the possible methods of suppressing the transition of the flow in the boundary layer from laminar to turbulent is to remove mass from that region through pores or slits in the wall. Studies have been under way for some years to develop a laminar flow control system mainly for the purpose of reducing the drag [1].

An important consideration in the design for such a profile is the geometry and configuration of the outlets through which the suction is effected.

Along that line, Klaus Gersten in [2] studied the three-dimensional incompressible laminar boundary layer flow along a plane wall when a slightly sinusoidal suction velocity distribution is applied normal to the direction of flow.

However, in recent years, the effects of magnetic fields on fluid motions in general, and on boundary layer flows in particular, have been widely discussed owing to their various applications in technological fields. We here refer only to the work published by Schlichting [3], Stuart [4] in which they studied the laminar flow along a plane wall with a constant suction velocity, and Lighthill [5] in which he studied the time-dependent viscous flow problem dealing with the effect of unsteady fluctuations of the free-stream velocity on the flow in the boundary layer of an incompressible fluid past two-dimensional bodies.

In the present paper we extend the works of both Schlichting and Klaus to investigate the effects of a uniform magnetic field on the steady boundary layer flow of an incompressible Newtonian fluid past a plane infinite wall when a transverse suction velocity distribution exists at the wall. The magnetic Reynolds number, \( Re_B = \mu_0 \sigma U_\infty \ll 1 \) (where \( \mu_0 \) is the permeability of the vacuum and \( \sigma \) is the electrical conductivity), is assumed to be small, so that the induced magnetic field is assumed to be small compared to the external magnetic field and can be neglected.

Using the method of perturbation, the velocity field and the temperature distribution in the boundary layer are obtained in Sections 3 and 4. In Section 5 the effects of the magnetic parameter on the shear stress and heat transfer coefficient at the wall are investigated. It was found that the shear stress in the direction of the mean flow and heat transfer increases while the transverse shear stress decreases as the magnetic parameter increases.

2. MATHEMATICAL ANALYSIS

Let us consider a uniform flow \( U_\infty \) past an infinite flat wall \( y = 0 \), with the x-axis on the wall parallel to the direction of \( U_\infty \) (see Figure 1).

\[ \text{Fig. 1.} \]
Assume that a suction velocity distribution of the form
\[ v(r) = -v_0 \left(1 + \varepsilon \cos \frac{\pi z}{I} \right) \]  
(2.1)
is applied to the wall. In (2.1), \( v_0 > 0 \), \( I \) is the wavelength and the amplitude \( \varepsilon \ll 1 \). Also assume that the generated flow is in the presence of a small uniform magnetic field applied in the direction of the \( y \)-axis.

It is evident that a suction velocity distribution, as given by (2.1), will lead to a cross-flow, and although the velocity and temperature fields will be independent of \( x \), the flow field itself will be three-dimensional. Therefore, for the flow region adjacent to the wall, the equation of continuity takes the form
\[ \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0. \]  
(2.2)

Also in the absence of any electric field, the Navier–Stokes equations are [6]:
\[ (V \cdot V)V = \frac{1}{\rho} \nabla P + \nu \nabla^2 V + \frac{\sigma}{\rho} (V \times B) \times B \]  
(2.3)
where \( \sigma \) is the electric conductivity of the fluid with \( \rho \) its density and \( \nu = \mu / \rho \) its kinematic viscosity. The last term in this equation is due to the presence of the magnetic field. It is known that when the magnetic Reynolds number \( R_e_B \) is small, the induced magnetic field can be neglected and \( B \) can be treated as constant, equal to the applied field. We also have, from the ideal uniform flow \( U_\infty \) past the wall,
\[ \frac{1}{\rho} \nabla P = \frac{\sigma}{\rho} (U_\infty \times B) \times B. \]  
(2.4)

Therefore, the components of the Navier–Stokes equations reduce to
\[ V \frac{\partial u}{\partial y} + W \frac{\partial u}{\partial z} = \nu \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) + \frac{\sigma B^2}{\rho} (U_\infty - u), \]  
(2.5)
\[ V \frac{\partial V}{\partial y} + W \frac{\partial V}{\partial z} = \nu \left( \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial P}{\partial y}, \]  
(2.6)
\[ V \frac{\partial W}{\partial y} + W \frac{\partial W}{\partial z} = \nu \left( \frac{\partial^2 W}{\partial y^2} + \frac{\partial^2 W}{\partial z^2} \right) - \frac{1}{\rho} \frac{\partial P}{\partial z} - \frac{\sigma B^2}{\rho} W. \]  
(2.7)

Finally, when the energy dissipation due to viscosity is neglected, the energy equation takes the form:
\[ V \frac{\partial T}{\partial y} + W \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right), \]  
(2.8)
where \( \alpha \) is the thermal diffusivity of the fluid. For the boundary conditions we have
\[ u = 0, \quad V = -V_0 \left(1 + \varepsilon \cos \frac{\pi z}{I} \right), \]  
\[ W = 0, \quad T = T_\infty \]  
(2.9)
at \( y = 0 \).
\[ u = U_\infty, \quad V = -V_0, \quad W = 0, \quad P = P_\infty, \quad T = T_\infty \]  
(2.10)as \( y \to \infty \).

For a strong suction, \( \varepsilon = 0 \), the solution can easily be obtained in the form
\[ \frac{u}{U_\infty} = (1 - e^{-y}), \quad V = -V_0, \quad W = 0, \quad P = P_\infty, \]  
(2.11)
\[ \theta = \frac{T - T_\infty}{\theta_\infty - T_\infty} = 1 - e^{-(V_0/\nu) y}, \]  
(2.12)
where
\[ \alpha = \left( \frac{V_0}{2\nu} + \sqrt{\frac{V_0^2}{2\nu} + M^2} \right). \]  
(2.13)
and \( M^2 = \sigma B^2 / \rho v \) is the magnetic field parameter.

When \( \varepsilon \neq 0 \), the solution of the problem is obtained by the perturbation method in the solution at \( \varepsilon = 0 \). We assume then that the solution is in the form:
\[ u = U_\infty (1 - e^{-y}) + u_1(y, z), \quad \cdots, \]  
\[ V = -V_0 + V_1(y, z) + \cdots, \]  
\[ W = w_1(y, z) + \cdots, \]  
\[ P = P_\infty + P_1(y, z) + \cdots, \]  
\[ \theta = (1 - e^{-(V_0/\nu) y}) + \theta_1(y, z) + \cdots. \]  
(2.14)
Substituting (2.14) in (2.2), (2.5)–(2.8) and equating the coefficients of different powers of \( \varepsilon \), we obtain the perturbation equations up to \( O(\varepsilon) \) as:
\[ V_1 \frac{\partial V_1}{\partial y} + W_1 \frac{\partial W_1}{\partial z} = 0, \]  
(2.15)
\[ -V_0 \frac{\partial u_1}{\partial y} + V_1 \frac{\partial u_1}{\partial y} + \alpha \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} = -\nu^2 u_1, \]  
(2.16)
\[ -V_0 \frac{\partial V_1}{\partial y} + V_1 \frac{\partial V_1}{\partial y} + \alpha \frac{\partial^2 V_1}{\partial y^2} + \frac{\partial^2 V_1}{\partial z^2} = -\nu^2 V_1, \]  
(2.17)
\[ V_0 \frac{\partial W_1}{\partial y} + V_1 \frac{\partial W_1}{\partial y} + \alpha \frac{\partial^2 W_1}{\partial y^2} + \frac{\partial^2 W_1}{\partial z^2} = -\nu^2 W_1, \]  
(2.18)
\[ -V_0 \frac{\partial \theta_1}{\partial y} + V_1 \frac{\partial \theta_1}{\partial y} + \alpha \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} = -\frac{\partial \theta_1}{\partial y} \]  
(2.19)
This is a set of linear differential equations for the velocity and temperature perturbations. Up to the same order of \( \varepsilon \) the boundary conditions are:
\[ u_1 = W_1 = \theta_1 = 0, \quad V_1 = -V_0 \cos \frac{\pi z}{I} \]  
(2.20)on \( y = 0 \),
\[ u_1 = W_1 = \theta_1 = 0 \]  
(2.21)as \( y \to \infty \).

3. THE CROSS-FLOW

We start first with finding the cross-flows \( V_1(y, z), W_1(y, z) \) and \( P_1(y, z) \) by solving equations (2.15), (2.17), (2.18), since