DIGITAL PHASE SHIFTERS IN AUTOMATIC FREQUENCY
CONTROL SYSTEMS OF QUANTUM FREQUENCY STANDARDS

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Methods are discussed for improving the metrological characteristics of quantum frequency standards and, in particular, their accuracy. The application of digital phase shifters in frequency conversion circuits is considered. It is shown that improvement of the reference signal spectrum of the AFC system makes it possible to increase the frequency standard accuracy by one to two orders of magnitude without affecting the output signal stability.

It is known [1] that the side spectrum of the frequency synthesizer signal in quantum frequency standards (QFSs) affects their short-term and long-term frequency stability. Accordingly, improvement of the spectral purity of frequency synthesizer signals is rightly assumed to be an important method of improving the metrological characteristics of QFSs.

Since only digital frequency synthesizers (DFSs) are used in modern QFSs Ch1-75 and Ch1-76, the spectrum of their signals can be improved primarily by suppressing low-frequency discrete components (fractionization noise) due to parasitic phase modulation of the output pulses of frequency dividers with a variable-fractional division factor (DVFDFs) [2].

Another way to control discrete side spectra is to replace DVFDFs with more efficient frequency converters, e.g., with digital phase shifters (DPSs) whose performance as smooth frequency control devices (without abrupt frequency changes typical of DVFDFs) was discussed in [3]. Unfortunately, pulse-position automatic frequency control (PP AFC) circuits with DPSs also suffer from low-frequency discrete (phase quantization) noise introduced by DPSs. Because of this, it is interesting to compare the performance of PP AFC systems with DPSs and PP AFC systems with DVFDFs.

Since the efficiency of the compared PP AFC systems is estimated from the degree of suppression of noise components introduced into the loop, the problem can be solved if the spectrum of the output oscillations of the pulse-position detector (PPD), which detects these oscillations, is known. However, the solution is complicated by the fact that problems associated with the calculation of the phase quantization noise spectrum of the PP AFC system oscillations are not adequately reflected in literature, especially in [4].

In this paper we have derived certain simple (acceptably accurate) and easily programmable analytical expressions for the spectral characteristics of phase quantization noise and used them to estimate the efficiency of PP AFC with DPSs and DVFDFs.

Let us find the spectrum of noise components in the PPD output signal of a PP AFC system with a DPS in the feedback loop (Fig. 1). In the DPS used in this circuit, the frequency-shifted signal is generated because of successive addition in time of high-frequency \( f = 1/T_0 \) oscillations (pulses) in the order of their increasing (decreasing) phases. If \( N \) is the number of quantization phases of the input high-frequency pulse signal (Fig. 2a), an example of a typical DPS signal is provided by a sequence (Fig. 2b) of high-frequency pulse packets shifted between each other in time by \( \tau = \frac{N}{i} T_0 \). If the number of pulses in the \( N \)-th packet of such a sequence is reduced by one, the average frequency of high-frequency pulses in a period \( T = 1/F \) of the low-frequency signal is shifted by

\[
\Delta f_{sh} = \frac{(T - 1)}{T} \Delta f = \frac{F - 1}{F}.
\]
Fig. 1. PP AFC loop with a digital phase shifter in the feedback loop: DPS) digital phase shifter, LPF) low-pass filter, VCO) voltage-controlled oscillator.

Fig. 2. Pulse signals at input 1 of the digital phase shifter (a), at inputs 2 and 3 and output 4 of the pulse-position detector (b, c, and d respectively).

If a uniform sequence of pulses of a reference frequency \( f_{\text{ref}} = 1/T_{\text{ref}} = f_{\text{sh}} \) is applied to one input of the PPD (RS flip-flop) (Fig. 2c) and a periodic sequence of period \( T \) of the DPS output pulses (Fig. 2b), nonuniformly distributed within this period, is applied to the second input, the PPD output is a sequence of width-modulated pulses with a period \( T \) (Fig. 2d) that can be represented by

\[
f(t) = \begin{cases} 
\Delta E & \text{if } n \frac{T}{T_0} \pm 1 < t \leq (n+1) \frac{T}{T_0} + \tau_0 + \Delta T_n, \\
0 & \text{otherwise}
\end{cases}
\]

where \( \Delta E \) is the amplitude of the PPD output pulses, \( \tau_0 \) and \( \Delta T_n (\Delta T_{n+1}) \) are respectively the initial and variable time shifts between the pulse sequences compared in the PPD, and \( n \) is a number of the natural series \( 0 < n < \frac{T}{T_0} \) equal to the number of the high-frequency pulse in the interval \( T \).

In general, as seen in Fig. 1, the relation between the shift \( \Delta T \) and the phase difference between the compared PPD input signals is

\[
\Delta T = \frac{\varphi_{\text{in}} - \varphi_{\text{ref}}}{2\pi} \quad T_{\text{ref}} = \frac{\varphi_{\text{in}} - \varphi_{\text{ref}}}{2\pi} \quad \text{for } f = f_{\text{sh}}
\]

where

\[
\varphi_{\text{in}} = \frac{2\pi}{N} \sum_{i=1}^{N} \sigma(t-iT/N)
\]

\[
\varphi_{\text{ref}} = 2\pi f t
\]

are the phases of the input and reference PPD signals respectively, and \( \sigma(t) \) is a unit function. The noise (parasitic) component of \( \varphi_{\text{in}} \) is