LINEAR MEASUREMENTS

ERROR IN THE MEDIAN CONTROL AND TRIMMING METHOD
IN CENTERLESS GRINDING OF TAPERED ROLLERS

L. A. Boguslavskii and S. S. Volosov

Translated from Izmeritel'naya Tekhnika, No. 10, pp. 13-16, October, 1963

The provision of machine tools with trimming devices is one of the methods for raising their precision and efficiency. An essential condition for using these devices consists of including, with a certain margin, their error in the tolerance field of the manufactured article. The median trimming method \cite{1,2} deserves the greatest attention among all the existing methods, since, with this method, gross errors have the least effect on the measurement results.

For \( n \) measurements \( x_1, x_2, x_3, \ldots, x_n \), the mean value is

\[
\bar{X}_1 = \frac{x_i}{n}.
\]

Their median value is

\[
\begin{align*}
X_{m_1} &= x_{(k+1)} \text{ for } n = 2k+1, \\
X_{m_1} &= \frac{x_k + x_{(k+1)}}{2} \text{ for } n = 2k.
\end{align*}
\]

We then have \( \bar{X}_1 \approx X_{m_1} \).

For the same \( n \) measurements, \( x_1, \hat{x}_2, \hat{x}_3, \ldots, \hat{x}_k, \ldots, \hat{x}_n \) for \( \hat{x}_L \gg x_L \), the mean value is

\[
\bar{X}_2 = \frac{\sum x_i}{n}.
\]

Their median value is

\[
\begin{align*}
X_{m_2} &= x_{(k+1)} \text{ for } n = 2k+1, \\
X_{m_2} &= \frac{x_k + x_{(k+1)}}{2} \text{ for } n = 2k.
\end{align*}
\]

We then find that

\( X_{m_1} \approx X_{m_2}; \quad n \rightarrow \infty \bar{X}_2 \approx \bar{X}_1. \)

Since the median value corresponds to the center of distribution of the dimensions' aggregate, i.e., \( p(X > \text{Me}) = p(X < \text{Me}) \) or \( \text{Me} = X \), where \( X \) is the value of the dimension which divides the aggregate into two parts \( S' \) and \( S'' \) of the same volume \( n/2 \). Moreover, \( S' \) includes all the values of \( x^n < \text{Me} \), and \( S'' \) all the values of \( x^n > \text{Me} \). Hence, individual variations of dimensions do not affect the median value, whereas the gross deviation of a dimension varies considerably the mean value.

The mean square error which characterizes in mean value trimming the instantaneous dispersion in the positions of the samples' dimension group centers amounts to:

\[
\sigma^2_x = D(\bar{X}) = \frac{1}{n} \frac{\sigma^2}{n} \quad \text{and} \quad \sigma_x = \frac{\sigma}{\sqrt{n}},
\]

where \( n \) is the number of components in a sampling, \( D(\bar{X}) \) is the distribution dispersion.
In trimming by means of quantile sampling, we find from [3] that:

$$\sigma_p = \sqrt{\frac{p(1-p)}{\varphi(x)}}$$

where $\sigma_p$ is the quadratic deviation of the quantile sampling, $\varphi(x)$ is the distribution law of the component dimensions' aggregate.

From the definition of a median we have $F(\text{Me}) = \frac{1}{2}$, where $F(\text{Me})$ is the law of integral distribution.

For a differential law of "normal" distribution, the "normal" curve is symmetrical with respect to the center of distribution and $\xi = a$, where $a$ is the theoretical mean. Under these conditions, $\varphi(x) = 1/\sigma \cdot \sqrt{2\pi}$. In a median-value trimming method, the mean square error will have the form

$$\sigma_m = \sqrt{\frac{1}{n} \sum \frac{1}{2}} \cdot \frac{1}{2} = \frac{\sigma}{\sqrt{n}} \cdot \sqrt{\frac{\pi}{2}}$$

It is easy to attain the same value for the mean square error as in mean-value trimming if the number of components in a sampling is increased by a factor of $\pi/2$.

It is possible to use in median-dimension control sampling ordinary double-limit electrical-contact transducers in a relatively simple circuit described in [1] without any special auxiliary equipment which is required for inductive, capacitive, or pneumatic transducers. The electrical-contact transducers are reliable in operation, simple to service, and provide errors of the order of 1.5-2 $\mu$, which are completely satisfactory for trimming systems. However, it should be noted that in order to obtain this degree of precision, the measuring device of the trimmer should control the components in a static condition; in other words, the components must come to rest in the measuring position.

The precision of grinding was tested on a centerless machine equipped with a trimming device and a measuring instrument at its final and finishing stages in grinding tapered rollers.

The accuracy diagrams plotted for the finished grinding of rollers without trimming display a tendency for a rising size of the articles in the course of grinding, thus showing that the wear of the grinding wheel is the prevailing factor. The diagrams of the finished grinding of rollers with a trimmer indicate that the latter keeps the roller dimensions within tolerances (Fig. 1).

Let us now examine how the location of the component in the measuring position of the automatic machine affects the trimming error. Such an error may be produced by placing the component for measurements in a skew position due to the penetration of dust or dirt onto the measuring knife-edges, thus affecting the operation of the electrical-contact transducer and producing gross deviations from the set dimension.

The location of the component in the measuring position can also result in a skew due to inadequate clamping (Fig. 2). Considering the smallness of angle $\alpha$, we then obtain