Gaussian beam propagation in dissipative solids

By Giacomo Caviglia, Dept of Mathematics, Via L.B. Alberti 4, I-16132 Genova and Angelo Morro, DIBE, Via Opera Pia 11a, I-16145 Genova, Italy (Fax: (010) 353-2777)

1. Introduction

Both the theoretical and practical interest of the subject motivate the wide attention to the wave properties of acoustic Gaussian beams. In particular, many works have been devoted to the scattering of acoustic Gaussian beams from a fluid-solid interface. Particular topics investigated in this framework are the lateral displacement of the reflected beam and the Rayleigh waves produced by the incident beam.

Also because of the nature of the problems under consideration, much research has been developed in connection with reflection of two-dimensional beams in a scalar-valued unknown. Pertinent references on the subject are the book by Brekhovskikh [1] and the papers by Breazeale et al. [2], Ngoc and Mayer [3], Claeys and Leroy [4], Pott and Harris [5], and Nagy et al. [6]. Still with a single unknown function, the paper by Choi and Harris [7] investigates the scattering of a beam from a curved interface. The paper by Felsen et al. [8] deals with two scalar-valued unknown potentials while a paper by Romeo [9] involves three potentials but is based on a numerical analysis. To our mind the use of potentials, instead of the displacement field, is scarcely operative in boundary-value problems.

Lately beams have been considered to describe wave propagation in heterogeneous media. To simplify the approach, appropriate models or approximations are applied. Essentially, though, they are based on ray propagation and follow asymptotic approximations, cf. [10–13].

The purpose of this paper is to investigate the propagation of a (Gaussian) beam. We let the framework be three-dimensional and the pertinent field be vectorial. The approach is general and is based crucially on the linearity of the model of material behaviour. Though it involves a mechanical context, namely viscoelastic (and, in particular, elastic) solids, we believe that, by following a similar approach, previous investigations about electromagnetic beams [14] may find a far-reaching generalization.
We consider the problem of wave propagation induced by data on the displacement and/or its normal derivative at a plane surface. Motivated by such a problem and because of the assumed linearity of the model, we investigate the propagation of time-harmonic waves. Significant developments are performed for a heterogeneous solid but definite closed-form expressions are established by letting the solid be homogeneous. The starting point of the present procedure is the description of the solid in terms of the stress-displacement. Then the evolution of the angular spectrum is examined as a function of the coordinate orthogonal to the plane surface. The approach is based on the use of the eigenvectors of a suitable problem; in a sense we parallel a method that traces back to Stroh [15] and Bazer and Burridge [16]. Among the improvements of the present approach is the account of dissipativity, a topic which is conceptually beyond the framework of previous investigations. Next, to obtain more definite conclusions, we restrict attention to Gaussian beams. In particular, in the approximation of wide beams we are able to determine the inverse Fourier transform and to show the widening, the spatial dispersion, and the attenuation of the beam, in terms of the material parameters of the solid.

2. The angular spectrum of plane waves

The pertinent field \( u \) is taken to be time-harmonic. Then for convenience we write \( u = U(x, y, z; \omega) \exp(-i\omega t) \) and, to fix ideas, we let \( \omega > 0 \). The physical (real) field is in fact a suitable superposition of such time-harmonic fields as we show in §5. For formal simplicity, we omit writing \( \omega \) and let \( U(x, y, z) \) stand for \( U(x, y, z; \omega) \).

The field \( u \) is given at a plane, say \( z = 0 \), along with the normal derivative \( u' = \partial u / \partial z \). Denote by \( U_0(x, y) \) and \( U'_0(x, y) \) the known fields at the plane \( z = 0 \). Our goal is to determine the field \( U(x, y, z) \) at any point \( (x, y, z) \) in space.

Assume that \( U_0(x, y) \) and \( U'_0(x, y) \) are Fourier-transformable with respect to \( x \) and \( y \) and consider

\[
\hat{U}_0(k_x, k_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} U_0(x, y) \exp[-i(k_xx + k_yy)] \, dx \, dy
\]

and similarly for \( U'_0(x, y) \). If the double Fourier transform of \( \hat{U}(k_x, k_y, z) \) is known at any plane \( z = \text{constant} \), then the solution \( U(x, y, z) \) is given by the inverse Fourier transform

\[
U(x, y, z) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{U}(k_x, k_y, z) \exp[i(k_xx + k_yy)] \, dk_x \, dk_y. \quad (2.1)
\]