REDUCTION OF THE FREQUENCY ERROR IN SINGLE-PHASE ELECTRODYNAMIC PHASE METERS

(E. S. Polishchuk and V. A. Khomyak)

Translated from Izmeritel'naya Tekhnika, No. 6, pp. 33-35, June, 1966

Variations in phase-meter readings for frequency deviations of ±10% from their normal values should not exceed according to GOST 8039-60 the numerical value of the instrument's precision class. Variations in the readings of single-phase electrodynamic phase meters due to frequency changes attain large values, owing to the phase-shifting circuits of these meters [1].

In treble-winding single-phase electrodynamic phase meters intended for audio-frequency circuits, the readings variation for frequency deviations of ±10% does not exceed 0.45° [2], and their frequency error is normalized according to the above-mentioned standard.

The utilization of a single-phase high-precision phase meter circuit at the commercial frequency of 50±5 Hz does not produce the required results, since the frequency error in treble-winding phase meters is small only if the phase-difference angles Θ_L and Θ_C approach in their absolute value π/2.

Angle Θ_L is equal at 50 Hz in the most advantageous case to 75-78°, owing to the considerable resistance of the choke and the resistance of the winding. The frequency error then increases noticeably. For instance, the readings of a phase meter type D578, class 0.5, with a scale of 180° and without frequency compensation vary by more than 2° (about 1.2%) for frequency changes of ±10%.

Figure 1 shows an improved circuit (as compared with that of instrument D578) of a single-phase treble-winding electrodynamic phase meter whose error does not exceed 0.45% for deviations of the nominal frequency of 50 Hz by ±10%. The proposed circuit has the following particular features.

1. The resistance of the split-coil circuits is equal to

\[ R_L = R_C = R = \sqrt{\frac{L}{C}} = \omega_0 L, \]  

moreover, as in instrument D578, the following equality also holds

\[ \omega_0 L = \frac{1}{\omega_0 C}, \]

where \( \omega_0 \) is the nominal phase-meter frequency.

2. Correcting network \( R_LC_C \) is connected in series with the split coil. The parameters of this network are equal respectively to

\[ R_C = R_L = R; \quad L_C = L; \quad C_C = C. \]

In order to analyze the frequency error of the phase meter shown in Fig. 1 let us derive the basic relationship between deviation angle \( \alpha \) of the moving part, the measured phase-difference angle \( \varphi \) and the parameters of the treble-winding phase meter. According to (2) this relationship can be written in the form

\[ \alpha = \arctan \frac{k_2 I_2}{k_1 I_1} \cdot \frac{\cos (\gamma - \varphi)}{\cos \varphi}, \]

where \( k_1 \) and \( k_2 \) are the sensitivities of the phase-meter coils.
where \( k_1 \) and \( k_2 \) are constant coefficients which depend on the dimensions of the moving and stationary coils of the instrument; \( I_b \) and \( I_1 \) are respectively the geometrical difference of currents in the split windings and the current in the unsplit coil whose torque is proportional to the difference of currents \( T_2 = I_C - I_L \); owing to the opposing connection of the split windings; \( \gamma \) is the phase angle between voltage \( U \) and current \( I_2 \) applied to the phase meter.

Let us evaluate the relation to frequency of the circuit parameters figuring in (4).

The unsplit coil circuit consists of a pure resistance and, therefore, the value of current \( I_1 \) does not depend on frequency. It can be easily shown that, provided conditions (1) and (2) are met, current \( I_2 \) in a treble-winding phase meter without a correcting network will be also independent of frequency and equal to

\[
|I_2| = \frac{U}{R}.
\]  

In fact

\[
\bar{I}_2 = \bar{I}_c - \bar{I}_L = \bar{U} \frac{\omega L^2 - \frac{1}{\omega^2 C^2} + j \left( \frac{\omega L}{\omega C} + \frac{1}{\omega C} \right)}{R \left[ 4R^2 + \left( \frac{\omega L}{\omega C} - \frac{1}{\omega C} \right)^2 \right]}. \tag{6}
\]

and the absolute value of current \( I_2 \) is found by taking condition (1) into consideration to be equal to

\[
|I_2| = \frac{U}{R} \sqrt{\frac{\omega L}{\omega C} - \frac{1}{\omega C}} = \frac{U}{R} \approx \frac{U}{R}.
\]

When the frequency deviates from its normal value, only angle \( \gamma \) changes. Its variation \( \Delta \gamma_1 \) evaluated from (6) with (1) taken into consideration can be represented as

\[
\Delta \gamma_1 = \tan^{-1} \left( \frac{\omega L}{\omega C} - \frac{1}{\omega C} \right) \approx \left( \frac{k - \frac{1}{k}}{2R} \right) \approx -\frac{k}{2} \text{[rad]}, \tag{7}
\]

where \( k = \frac{\omega}{\omega_0} \).

A deviation of the frequency from its nominal value by \( \pm 10\% \) leads according to (7) to a change in angle \( \gamma \) of \( \pm 5.4^\circ \). Such a variation of the angle will obviously lead to considerable frequency errors.

Variations of angle \( \gamma \) due to frequency changes can be reduced to an insignificant value by incorporating an RLC correcting network in the treble-winding phase meter. In fact, the frequency deviation in a circuit with a correcting network leads to a phase angle change \( \Delta \gamma_1 \) between current \( I_2 \) and voltage \( U_0 \). At the same time a certain phase difference \( \psi \) arises between voltages \( U_0 \) and \( U \) which are applied to the phase meter. If angles \( \Delta \gamma_1 \) and \( \psi \) are equal, the phase difference between current \( I_2 \) and voltage \( U \) remains unchanged.

By writing down an expression for the impedance of the split-coil circuit and separating its real and imaginary parts, we find after appropriate transformations by taking conditions (1-3) into consideration an expression for the phase difference angle \( \psi \):

\[
\text{Fig. 1}
\]