EVALUATION OF THE OPTIMUM FREQUENCY OF A PARAMAGNETIC-FIELDS SCANNING CONVERTER

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Space characteristics of paramagnetic fields are evaluated by means of scanning conversion [1, 2] in which a field function continuous in the space of its parameter and in time is subjected to discretization with respect to time. Let us examine devices with pulse-modulation in which the sensing element (SE) used for converting the parameter value into an electrical signal periodically "scans" the entire space, so that trains of measuring pulses appear at its output. The information parameter of such an electrical signal consists of the time difference between the measured and reference pulses. The electrical signal is amplified (Fig. 1) and fed to a threshold device which fixes the position of the pulse with respect to time. It is then subjected to pulse demodulation for determining the time difference between the measured and the reference pulses. The demodulator's output signal corresponds in a certain scale to the field point coordinate for a given parameter value.

Scanning conversion entails two errors depending on the scanning frequency \( f_{sc} \). The first error \( \delta_d \) (Fig. 2) is due to the discretization of the continuous input function, and it rises with \( f_{sc} \) [3]. The second error is due to the sensing element's internal noise, which occurs, for instance, in analyzing luminous fields by means of photoelectric transducers. The noises produce a demodulation error \( \delta_{dm} \) which is proportional to the scanning frequency. Thus, it is possible to find an optimum scanning frequency \( f_{sc, opt} \) for which the total error is at a minimum.

\[
\epsilon^2 = \delta_d^2 + \delta_{dm}^2,
\]

Let us evaluate errors \( \delta_d \) and \( \delta_{dm} \).

Without limiting the generality of the problem let us examine its application to single-coordinate fields which are represented by a two-dimensional function (Fig. 3):

\[
B(x; t) = B(x_0 + x + v_x t),
\]

where coordinate \( x \) is in the range of \( 0 \leq x \leq X_{\text{max}} \); \( B(x) \) is a local parameter-distribution function which exists in the range of \( \pm X_{\text{max}}/2 \) and represents the "relief" of the field; \( v_x \) is the velocity of the relief displacement.

Let the scanning speed be \( v_{sc} = X_{\text{max}}f_{sc} \gg v_x \). Space filtration of the input function occurs in scanning conversion owing to the finite dimension of the sensing elements. If we denote the position of the sensing element by coordinate \( \varepsilon \) and its weight function along coordinate \( x \) as \( B(x) \) the energy flux received by the element will be equal to

\[
\Phi (\varepsilon; t) = b_y \int_{-\frac{\alpha}{2}}^{\frac{\alpha}{2}} B(x; t) + b(x) \, dx,
\]

Fig. 1. 1-parametric field; 2-sensing element which converts parameters into electrical signals; 3-unit for sweeping (scanning) the parametric field; 4-amplifying and converting unit; 5-threshold unit adjusted to a given parameter value; 6-pulse modulator which measures time intervals between pulses; 7-recording device.
Fig. 2. Relationship of the discretization, demodulation and total errors to the scanning frequency.

Electrical signals at the output of the sensing element consist of a train of pulses modulated with respect to their position in time and their duration. Assuming that $x_d \ll X_{\text{max}}$, i.e., that $\tau_p \approx x_d / v_{\text{sc}} \ll \tau_{\text{sc}} \approx 1/f_{\text{sc}}$, we find that the spectrum of the pulse train can be represented by the a single-pulse spectrum with an envelope of

$$u_s(t) = k_{\text{SE}} \Phi \left[ v_{\text{sc}} t \left( 1 + \frac{v_x}{v_{\text{sc}}} \right) \right] \approx k_{\text{SE}} \Phi (v_{\text{sc}} t),$$

where $k_{\text{SE}}$ is the sensing element's conversion factor.

Function $\Phi (e, t)$ is a two-dimensional function and, therefore, it can be represented by two elementary spectra consisting of a spatial frequency spectrum $S_{\Phi}(\nu)$ and the time frequency spectrum $S_{\Phi}(f)$, which are determined by the character of function $\Phi(e,t)$ in space and time respectively. According to the rules of the Fourier multidimensional transformation [5] we obtain

$$S_{\Phi}(f) = \int_0^\infty \Phi(e, t) e^{-j2\pi ft} dt = \int_0^\infty B(x, t) e^{-j2\pi ft} dt,$$

$$S_{\Phi}(\nu) = \int_0^\infty \Phi(e, t) e^{-j2\pi \nu t} d\nu.$$

The replacement of $\Phi(e, t)$ by $B(x, t)$ in (3) is permissible, since conversion (2) refers only to the spatial argument of function $B(x, t)$.

Expression (3) can be used for evaluating error $\delta_d [3]$,

$$\delta_d^2 = \frac{3X_{\text{max}}}{f_{\text{sc}}} \int_0^\infty S_{\Phi}(f) df,$$

where $\delta_d^2$ is the variance of the absolute error.

Let us now deal with error $\delta_{\text{dm}}$.

The spatial characteristics of the field are detected by means of a threshold device whose operating level $U_{\text{th}}$ corresponds to a definite parameter value. In its preliminary amplification, signal $u_{\text{s, out}}(t)$ is distorted owing to the limited bandwidth of the amplifying channel and to the presence of noise at the input.

Let us denote (Fig. 4) the root-mean-square value of output pulse distortions by $\Delta U_{\text{ch}}$.

The pulse-modulation time error due to distortions in evaluating the position of the pulse by its leading edge amounts to [4]:

$$\Delta t = \frac{\sqrt{\Delta U_{\text{ch}}^2}}{k_{\text{mf}}},$$

where $k_{\text{mf}}$ is the steepness of the measured front of the pulse.