ACCOUNTING FOR THE EFFECT OF PRESSURE-GAUGE FEEDING CHANNELS IN THE PRESENCE OF A TEMPERATURE GRADIENT

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In a number of instances pressure is measured by means of a feeding channel. This is especially necessary when the temperature in the working volume of the gauge is above that permissible for its normal operation. In such a case a feeding channel is used, along whose length the temperature drops to the tolerated value (Fig. 1). The amplitude of periodic disturbances which may occur in the operating volume is distorted by the feeding channel and the cavity in front of the gauge membrane. The problem of pressure distortions contributed by the channel-cavity system has been solved by I. A. Charnyi [1] for liquid oscillations with friction taken into account. Moreover, the author considered the problem to be planar and the pressure and velocity disturbances $\delta P$ and $\delta V$ to be small; therefore he used linearized acoustical equations, assuming that the cavity in front of the membrane has a lumped cubic capacity. Thus, he neglected the wave processes occurring in that cavity, since its length is considerably smaller than the testing wavelength, and he considered the temperature along the channel to be constant. For the recording of pressures in a gas medium it is possible to make a few more assumptions. It is possible to neglect the elasticity of the tube walls and of the membrane as compared with that of the gas; it is also possible to neglect the reduction of amplitude due to acoustical energy losses produced by the gas force of friction against the channel walls. The effect of friction forces can be evaluated from the formula for the attenuation ratio [2]

$$\alpha = \frac{V_{av}}{d_c \omega},$$

where $\alpha$ is the attenuation ratio, $\omega$ is the angular velocity of forced oscillations, $d_c$ is the channel diameter, $a$ is the speed of sound in gas, $\nu$ is the kinematic viscosity.

For frequencies of the order of a kilohertz and a channel diameter of the order of several millimeters, it is possible to neglect friction in channels which are not too long.

Taking into consideration the assumptions made for a gas medium, the Charnyi formula becomes:

$$\lambda = \frac{\delta P_f}{\delta P_0} = \frac{l}{\cos \omega \frac{l}{a} - \omega \frac{v_0}{a} \sin \frac{\omega l}{a}},$$

where $\lambda$ is the distortion factor; $\delta P_f$ and $\delta P_0$ are the oscillation amplitudes respectively at the output and the input of the channel; $v_0$, $f$, and $l$ are the volume, cross-sectional area, and length of the channel.

A similar case has been examined in [3]. The authors considered the channel and its cavity as a system with lumped elasticity and mass, and obtained the following expression for the system's natural resonant frequency

$$\omega_c = \sqrt{\frac{K}{M}} = \frac{a}{\sqrt{\frac{2}{\pi} f \left( \frac{2}{\pi} f + \frac{v_0}{f} \right)}}.$$
Fig. 1. Schematic for connecting a transducer through a channel to the measured volume. 1) The working volume; 2) a channel of length \( l \) and cross-sectional area \( f \); 3) a cavity of volume \( V_0 \) in front of the gauge membrane; 4) membrane.

The distortion coefficient is then found from the formula

\[
\lambda = \frac{1}{1 - \left( \frac{\omega_p}{\omega_n} \right)^2}
\]

where \( \omega_p \) is the frequency of the process, \( \omega_n \) is the frequency of natural oscillations.

It has been noted above that the assumption of a constant temperature along the length of the channel does not always hold. Therefore, let us derive the distortion factor by taking into consideration the temperature gradient along the channel, but retaining the assumptions made in Charnyi's simplified formulas. Since the pressure and velocity disturbances are considered to be small in their absolute value, it is possible to neglect their effect on thermal processes, i.e., it is possible to consider that the distribution of temperature along the channel does not depend on wave processes. In order to find the distortion produced by the channel-cavity system, let us find the amplitude drop along the channel of forced sinusoidal oscillations fed to its input.

Let us write the linearized acoustical equations

\[
\begin{align*}
\frac{\partial^2 P(x,t)}{\partial t^2} + \rho(x) \frac{\partial P(x,t)}{\partial t} &= 0 \\
\frac{\partial^2 v(x,t)}{\partial t^2} + \rho(x) \frac{\partial v(x,t)}{\partial x} &= 0
\end{align*}
\]

where \( \rho(x) \) and \( v(x) \) are the undisturbed values respectively of the density and velocity of sound.

From a conversion of (1) we can obtain equations for evaluating \( \delta P \) and \( \delta v \)

\[
\begin{align*}
\frac{1}{a^2(x)} \cdot \frac{\partial^2 P(x,t)}{\partial t^2} &= \frac{1}{T(x)} \cdot \frac{\partial^2 T(x)}{\partial x^2} \\
\frac{\partial^2 v(x,t)}{\partial t^2} &= \frac{\partial^2 P(x,t)}{\partial x^2}
\end{align*}
\]

where \( T(x) \) is the undisturbed value of temperature in the given cross section.

The boundary conditions comprise periodic input-channel oscillations \( x = 0, \delta P(t) = \delta P_0 \omega t \). We find at the end of the channel, on the basis of a material balance, that (quantities of the second order of smallness are neglected):

\[
x = l; \delta v(l,t) \cdot \delta t = \delta P_0 \omega t
\]

where \( \delta P_0 \) and \( \delta t \) are respectively the density and velocity sound at the end of the channel.

We can finally write down the conditions at the end of the channel as

\[
x = l; \delta v(l,t) = A \frac{\partial P(l,t)}{\partial t}, \text{ where } A = \frac{\nu_0}{l^{-1} a_l^2}
\]