A Particle Picture of "Tunneling" and the Nature of "Photon"–Matter Interaction

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Brownian motion is employed as a model to formulate a probabilistic statement of the Heisenberg uncertainty relationship. By means of the latter, a tunneling formula identical to that obtained from the WBK method is derived from the particle picture. The nature of photon–matter interaction is discussed in the light of the quantal Brownian motion.

KEY WORDS: Tunneling; Poisson process; uncertainty principle; photon.

1. INTRODUCTION

A derivation of the Schrödinger equation from Newtonian mechanics has been eminently achieved by means of a statistical model. The electron is regarded as a point particle of mass m constantly undergoing a Brownian motion with the diffusion coefficient of \((\hbar/2m)\), where \(\hbar\) is the Planck’s constant divided by \(2\pi\). As a consequence of the stochastic model, we can interpret the uncertainty action \(s\), defined to be the product of the root mean square

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deviation of any canonical conjugate pair $\Delta q_i \cdot \Delta p_i$, as being a result of the quantal Brownian motion. Thus, from Einstein's equation of diffusion,

$$\langle (x - x_0)^2 \rangle = 2Dt$$

(1)

it can be easily shown that\(^{(1)}\)

$$\langle \Delta x \rangle \langle \Delta p \rangle = h = \bar{s}$$

(2)

upon replacing $D$ by $h/2m$. Since Eq. (2) is an equality, we may conjecture that $h$ is the statistical average of the uncertainty action $\bar{s}$.

A small particle performs Brownian motion in a fluid which is caused by collisions with molecules of the fluid. The probability of no collision in the time interval $(0, t)$ is given by

$$P_0(t) = e^{-t/\tau}$$

The random collision process is referred to as the Poisson process. Hence, in this paper we propose the hypothesis that the uncertainty action (of a canonical conjugate pair) of a particle in a quantal Brownian motion is again a Poisson process. Thus we have the following postulate:

Whatever the value of the uncertainty action $s$ of a particle is, for a small interval of length $\Delta s$ in the sample space the probability of finding the value of $s$ in $\Delta s$ is $\lambda \Delta s + O(\Delta s)$, where $\lambda > 0$ and $O(\Delta s)$ denotes a zero quantity which is of smaller order of magnitude than $\Delta s$, and $1 - \lambda \Delta s$ is the probability of not finding the value of $s$ in $\Delta s$.\(^{(2)}\)

With this postulate we shall formulate the probability density function $\rho(s)$ in the sample space $s = \{s : h/2 \leq s < \infty\}$. It goes without saying that the lower bound $s \geq h/2$ is specified in accordance with the Heisenberg uncertainty principle.

Since $\rho(s)$ is the probability density function, $\rho(s) \Delta s$ is the probability of finding the value of the uncertainty action of the particle in the interval $(s, s + \Delta s)$. Let us consider two contiguous intervals, $(h/2, s)$ and $(s, s + \Delta s)$ in the sample space. The value of the uncertainty action of the particle must not be in $(h/2, s)$ and be in $(s, s + \Delta s)$. Let $P_0(s)$ be the probability that the value of the uncertainty action of the particle is not in the interval $(h/2, s)$. From this conjecture and the aforementioned postulate, we obtain

$$\rho(s) \Delta s = P_0(s) \lambda \Delta s$$

(3)

and

$$P_0(s + \Delta s) = P_0(s)(1 - \lambda \Delta s)$$

(4)