AN IMPROVED THEORY OF MICROWAVE OPEN RESONATORS

Chenghe Xu, Lezhu Zhou, and Anshi Xu

Department of Radio-Electronics
Peking University, Beijing
The People's Republic of China

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The analytic theory of microwave open resonators is improved here by means of solving the complex Airy equation, that the Q-value as well as the argument of field profile may be given accurately. The resonance conditions for open cavities with high and moderate Q-values are derived analytically. Theoretical Q-values are well agreed with the measured ones from experiments.

Key words: microwave open resonators, wave propagation in waveguides, electron cyclotron masers, gyrotrons, radiation from open resonators.

INTRODUCTION

Microwave open resonators have been used greatly as the resonant system in gyrotrons, and its theory was developed to a large extent [1,2]. Several years before, we suggested an analytic theory which can give the resonant frequency and field profile well agreed with the experimental results[4]. However the Q-value calculated from the previous theory may be higher than the measured one even by 30-50%. The purpose of this article is to improve the solution of field equation in complex variables in order to get the imaginary part of complex frequency as well as the Q-value more accurately.
SOLUTION OF FIELD EQUATION OF COMPLEX VARIABLES

The microwave open resonator in the form of waveguide with slowly varying cross-sections can be operated with a single mode, where the field component of Hmn mode can be expressed as $f(z) \exp[i\omega t - ih(z)z]$ and the field function $f(z)$ satisfies the following equation [3]

$$f''(z) + h(w, z)f(z) = 0,$$ (1)

where $h(w, z)$ denotes the axial wavenumber, and for the conventional axi-symmetric cavities

$$h^2 = \omega^2/c^2 - \mu_{mn}^2/r^2(z)$$ (2)

where $r(z)$ denotes the radius of a certain cross-section at $z$, and $\mu_{mn}$ is the $n$-th zero of $J_m(x)$, Bessel function of the first kind of order $m$.

For finite large Q-value, the angular frequency

$$\omega = \omega_r + iw_r.$$ (3)

The real $\omega_r$ is the resonant frequency and the Q-value can be expressed by

$$Q = \omega_r/(2\omega_c).$$ (4)

We define the cutoff position $z_c$ by

$$h(z_c)|_{\omega_c=0} = 0,$$

therefore the radius of cutoff cross-section $r_c$:

$$r_c = r(z_c) = \mu_{mn}c/\omega_r.$$ (5)

In order to find the field solution of Eq. (1) in the neighborhood of $z_c$, the $h^2(z)$ is expanded into a Taylor series about the point $z_c$:

$$h^2(z) = h^2(z_c) + \left[h^2(z_c)\right]'(z-z_c) + \ldots = -\beta^2 T,$$ (6)

where $T = t - is$, $t = -\beta (z-z_c)$,

$$\beta^2 = \left[h^2(z_c)\right]' = 2\mu_{mn}^2 r'(z_c)/r_c^2,$$

and $s = ih^2(z_c)/\beta^2 = 2\omega_c\omega/(c^2\beta^2)$. (7)

It should be pointed out that $s$ is a quantity independent on $z$. Replacing $z$ by new variable $T$ as given in Eq. (7) and setting $F(T) = f(z)$, Eq. (1) can be transformed into

$$F''(T) - T F(T) = 0,$$ (8)