A UNIFIED APPROACH TO THE EFFECTS OF SYMMETRY AND PERIODICITY ON BOUNDARY-VALUE PROBLEMS

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ABSTRACT

A generalized approach to the effects of symmetry and periodicity on boundary-value problems is presented, especially as it pertains to the guiding structures encountered in electromagnetics. Several examples of structures and their dispersion behavior are given, and the theory predicts well what other authors claim.

1. INTRODUCTION

An important advantage to solving EM field problems using wave potentials is that the effects of symmetry on the wave expansions have been extensively studied [1-8]. Scalar wave expansions must be complete to represent any possible field where the expansions are used. It is not always necessary, however, to have the most general expression for the fields. The symmetries of the problem may eliminate some series terms of the wave expansion. Further, a simple boundary may uncouple some or all the series terms, or couple only selected terms.

It is the purpose of this paper to extend the previous treatments of symmetry and periodicity enough to handle a wide range of boundary-value problems; most of the analysis is based on studies by Cain [4] and Chen [5].
2. THE CLASSICAL TREATMENT OF SYMMETRY USING THREE-DIMENSIONAL FLOQUET ANALYSIS

2.1 Coordinate Independent Solution

Floquet's theorem is central to the classical treatments of symmetry. This theorem asserts that the fields at two locations are related if the boundary from the perspective of the two locations is indistinguishable. Further, the relation is one of a simple complex magnitude. In other words, the functional form of the fields is determined by the shape of the boundary; if the boundary looks the same at two points, the fields must also have the same form. Since the boundary does not determine the amplitude of the fields, it is not surprising that at points of symmetry, the fields can have different amplitudes while still retaining the same functional form.

Mathematically, Floquet's theorem can be given in terms of the wave potentials, since the fields are derived from these potentials. Thus, at any position \( \mathbf{R} \) in the general cylindrical coordinate system \((r, \theta, z)\), the wave potentials are related to the potentials displaced by a vector \( \mathbf{D} \) through the equation:

\[
\psi(\mathbf{R}) = e^{j \mathbf{\Gamma} \cdot \mathbf{D}} \psi(\mathbf{R} + \mathbf{D}) \tag{1}
\]

It is assumed that the displacement \( \mathbf{D} \) places the potentials to a point of symmetry. Since periodicity is a form of infinite symmetry, this equation holds true for any two points separated by a period all along a periodic system. \( \mathbf{\Gamma} \), in this case, then corresponds to a fundamental complex propagation constant of the system. If this constant were not to exist, only static fields could exist on a periodic structure, since the fields would be identical to any measure of periods apart. In a lossless system, \( \mathbf{\Gamma} \) is purely imaginary, or purely real [9].

The wave potentials are normally expanded into some type of Fourier sum. The fields are then derived from these sums. If the wave expansion is given in terms of sums expanded about the general cylindrical coordinate system,