Fokker–Planck and Langevin Descriptions of Fluctuations in Uniform Shear Flow

Rosalio F. Rodriguez, E. Salinas-Rodríguez, and James W. Dufty

Received January 3, 1983

The Boltzmann description of the preceding paper for tagged particle fluctuations in a nonequilibrium gas is further analyzed in the limit of small mass ratio between the gas and the tagged particles. For a large class of nonequilibrium states the Boltzmann–Lorentz collision operator for the tagged particle distribution is expanded to leading order in the mass ratio, resulting in a Fokker–Planck operator. The drift vector and diffusion tensor are calculated exactly for Maxwell molecules. The Fokker–Planck operator depends on the nonequilibrium state only through the hydrodynamic variables for the fluid. The diffusion tensor is a measure of the "noise" amplitude and is not simply determined from the nonequilibrium temperature; instead, it depends on the fluid stress tensor components as well. For the special case of uniform shear flow, the Fokker–Planck equation is of the linear type and may be solved exactly. The associated set of Langevin equations is also identified and used to describe spatial diffusion in the Lagrangian coordinates of the fluid. The effect of viscous heating on diffusion is discussed and the dependence of the diffusion coefficient on the shear rate is calculated.

KEY WORDS: Nonequilibrium fluctuations; Fokker–Planck equation; Langevin equation; shear flow; kinetic theory; diffusion.

1. INTRODUCTION

The equations for transport and fluctuations of a tagged particle in a nonequilibrium gas were described in the preceding paper. The kinetic equation for the tagged particle is characterized by a Boltzmann–Lorentz...
collision operator as a functional of the fluid distribution function, and the tagged particle was taken to be mechanically identical to the fluid particles. This description is extended here to the case of unequal masses for the tagged and fluid particles. Specifically, the tagged and fluid particles are assumed to interact via the same force law as holds between fluid particles, but the mass of the tagged particle, $M$, is considered to be large compared to that of the fluid particle, $m$. The mass ratio, $\epsilon = m/M$, is therefore a small parameter in terms of which the Boltzmann–Lorentz operator may be expanded. This expansion is carried out to leading order, $\epsilon^{1/2}$, resulting in a second-order differential operator of the Fokker–Planck form. The drift vector and diffusion tensor in this operator are calculated exactly for Maxwell molecules. A special feature of the Maxwell potential is that the Fokker–Planck operator depends on the nonequilibrium state of the gas only through low-order moments of the fluid distribution function, which can be identified in terms of the hydrodynamic variables and irreversible fluxes of the nonequilibrium gas. In particular, the diffusion tensor is proportional to the components of the pressure tensor. This implies that the "noise" in the dynamics of the tagged particle is not simply thermal as in equilibrium. The Langevin description associated with the Fokker–Planck equation is also identified to emphasize this difference between equilibrium and nonequilibrium fluctuations.

In the case of a fluid with uniform shear flow, the Fokker–Planck equation is of the "linear" type. The fluctuations and time correlation function for position and velocity of the tagged particle are easily determined from either the Fokker–Planck equation or the (linear) Langevin equations, in the same way as for equilibrium fluctuations. The results agree with those of Reference 1 obtained directly from the Boltzmann–Lorentz equation. An advantage of the Fokker–Planck description is that the distribution function and joint probability density for the tagged particle may be determined exactly, in contrast to the more general Boltzmann–Lorentz description for which only moments of these functions are tractable. Furthermore, since the "linear" Fokker–Planck equation corresponds to a Gaussian–Markovian process, all multipoint probability densities may be expressed in terms of the distribution function and joint probability density. This provides an example of a system far from equilibrium for which all statistical properties may be calculated exactly. The solutions to both the Fokker–Planck equation and the Langevin equations are given in Section 3. The general results are difficult to interpret owing to the combined effects of anisotropy induced by the shear field and the viscous heating. Consequently attention is focused on the reduced distribution function for spatial coordinates of the tagged particle, referred to the Lagrangian frame of the fluid. The diffusion coefficient is calculated as a function of the shear rate and the nonequilibrium temperature.