A numerical calculation is carried out by the finite-difference method based on proposed equations for a turbulent submerged jet containing an admixture of solid particles. The relative longitudinal particle velocity and the influence of particles on the turbulence intensity are taken into account. The calculated results adequately agree with available experimental data. A turbulent two-phase jet is examined in [1] on the basis of the theory for a variable density jet, assuming equal mean velocities for the gas and particles and not considering the influence of particles on the turbulence intensity. Particles are analogously taken into account by a noninertial gas mixture in [2, 3], and a particle Schmidt number of 1.1 is assumed in [4]. A model is proposed in [5] which takes into account the influence of particles on the turbulence intensity of the gas phase. Problems concerning the initial and main sections of a submerged jet were solved in [6] by the integral method on the basis of this model and the assumed equality of the mean velocities of the gas and particles. Turbulent mixing of homogeneous two-phase flows with allowance made for dynamic nonequilibrium of the phases is considered in [7]. However, the neglect of turbulent transfer of particle mass and momentum led to a physically unrealistic solution for the particle concentration in the far field of the mixture. A two-phase jet is considered in the present work on the basis of the theory of a two-velocity continuous medium [8, 9] with allowance made for turbulent transfer of particle mass and momentum. The influence of particles on the turbulence intensity of the gas phase is taken into account with the model of [5].

1. Formulation of the Problem

We examine plane or axisymmetric turbulent flow of a gas containing an admixture of solid spherical particles of the same diameter \( d \). The two-fluid theory of [8, 9] will be used to describe such a two-phase
flow. To apply this theory it is required that the mean distance between particles is small compared with the characteristic flow dimension. A more exact condition for applying this theory to two-phase turbulent flow is developed in [10]. We will assume that the volume occupied by particles can be neglected. In addition, we restrict ourselves to the case of an isobaric, incompressible carrying medium.

We direct the x axis along the symmetry axis of the jet and the y axis perpendicular to it, and let u and v be the corresponding velocity components. Representing all parameters in the form of the sum of mean and fluctuating components, we carry out the formal procedure of averaging the equations of the two-fluid theory [8, 9]. We apply the usual assumptions that the transverse velocity is much less than the longitudinal, and the y derivatives are much larger the x derivatives. The equations for the mean values, neglecting triple correlations, are

\[
\begin{align*}
\frac{\partial}{\partial x} yu + \frac{\partial}{\partial y} yv &= 0, \\
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{1}{y'} \frac{\partial}{\partial y} y' \langle u'v' \rangle &= f_x, \\
\frac{\partial u_s}{\partial x} + (\rho_s + \langle \rho_s' \rangle) \frac{\partial u}{\partial y} + \frac{1}{y'} \frac{\partial}{\partial y} y' \rho_s \langle u_s'v_s' \rangle &= -f_x, \\
\frac{\partial}{\partial x} y' \rho_s u_s + \frac{\partial}{\partial y} y' (\rho_s + \langle \rho_s' \rangle) &= 0
\end{align*}
\]

(1.1)

Here the subscript \( s \) refers to particle parameters; \( j = 0 \) and \( j = 1 \) correspond to plane and axisymmetric cases; primes denote fluctuating components; \( \rho_s \) is the particle density (mass of particles per unit volume); and \( f_x \) is the longitudinal component of the mean interaction force between phases. Equations (1.1) are dimensionless: the velocity is referenced to the maximum gas velocity \( V \) at the nozzle exit; the linear dimensions are referenced to the nozzle radius \( R \); the particle density is referenced to the gas density \( \rho \) so that \( \rho_s \) is the particle concentration.

According to (1.1), the particle motion is determined by two factors: the interaction force \( f_x \) between particles and gas and the apparent tangential stress \( \rho_s (u_s v_s') \), which physically arises from the transfer of particle momentum by velocity fluctuations of the gas.

By neglecting the fluctuating components in \( f_x \), we determine in the first approximation the interaction force directly from mean parameters in the form

\[
f_x = \beta \rho_s \langle u_s - u \rangle
\]

(1.2)

Here \( \rho_s \) is the density of the particle material; \( \mu \) is the gas viscosity; \( C_x \) is the drag coefficient of a particle; and the function \( G = G(Re) \) gives the difference of the particle drag law from the Stokes law, for which \( G = 1 \). It is assumed in (1.2) that the drag force is determined by the sum of the drags of separate particles with the condition that the flow past them is the average velocity at the given point. In fact, (1.2) is the usual expression for the drag force in a nonturbulent flow [11] and should be considered as an approximation. We note that the standard drag law for a sphere in an incompressible fluid [12] was used in the calculation.

It is necessary to indicate that equations for conservation of total momentum of particles and gas and of the mass of particles in turbulent flows are derived in a number of works [10, 13, 14]. An expression for the particle drag force analogous to (1.2) was used in [13].

For the equations of particle momentum in the transverse direction, we assume, in much the same way as in [7], equality of the mean transverse velocities of particles and gas: \( v_s = v \). For free turbulent flow this hypothesis can be expected to be fairly accurate.

To connect the correlations of the fluctuating components in (1.1) with mean parameters, we use the Boussinesq relations

\[
-\langle u'v' \rangle = \varepsilon \frac{\partial u}{\partial y}, \quad -\langle u_s'v_s' \rangle = \varepsilon_s \frac{\partial u_s}{\partial y}, \quad -\langle \rho_s'v_s' \rangle = \varepsilon_s \frac{\partial \rho_s}{\partial y}
\]

(1.3)

Here \( \varepsilon \) and \( \varepsilon_s \) are turbulent viscosity coefficients of the gas and particles, the particles being considered as a second continuous medium; in this case the difference in the turbulent transfer of momentum and