SOME RESULTS OF A NUMERICAL ANALYSIS OF
TRANSIENT WAVES IN GAS SUSPENSIONS

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Special aspects of the transmission of transient waves through gas mixtures carrying suspended solid particles of chemically inert substances are examined. The influence of the parameters of the gas suspension on the conditions governing the occurrence of transient processes is discussed. The interaction of shock waves with a dust-laden half-space is considered. The results of calculations relating to the decay of an arbitrary discontinuity during the reflection of a shock wave from a wall are presented.

Introduction

Many papers relating to wave motion in gases containing solid particles have recently been published [1-10]. The majority of these papers has been concerned with steady-state shock waves and has considered the equilibrium parameters behind the leading edges of the shock waves and the structures of the leading edges themselves. Some transient aspects of flows in gas suspensions were analyzed in [6-8]. In this paper we shall present the results of some numerical calculations regarding transient wave processes in mixtures of the gas-particle type carried out by integrating the equations of two-phase motion, using a different method of through calculation specially developed for this purpose.

1. Fundamental Equations

We shall study the flows of gas suspensions by the methods commonly employed in the mechanics of continuous media [9-11], subject to the following main assumptions: 1) the characteristic linear scales of the flows are much greater than the dimensions of the particles and the distances between them; 2) the particles are spherical and of a single size; 3) fractionation, collision, and coagulation of the particles are absent; and 4) the viscosities and thermal conductivities of the phases are only important in relation to interactions between them. The corresponding system of differential equations for the description of one-dimensional transient motions may be written in the following form:

\[
\begin{align*}
\frac{d\rho_1}{dt} + \rho_1 \frac{\partial u_1}{\partial x} &= 0, \\
\frac{d\rho_2}{dt} + \rho_2 \frac{\partial u_2}{\partial x} &= 0, \\
\rho_1 \frac{d^2u_1}{dt^2} + \rho_1 \frac{\partial p}{\partial x} &= n f, \\
\rho_2 \frac{d^2u_2}{dt^2} &= n q,
\end{align*}
\]

(1.1)

Here \(\rho_1, \rho_2, u_1, u_2, E_1, E_2\) are, respectively, the average and true densities, and the volumetric content, velocity, and specific internal and total energies of the i-th phase. The subscripts 1 and 2 relate to the parameters of the gas and the particles, \(p\) is the pressure of the mixture, and \(d\) and \(n\) are the diameter and number of the particles in unit volume. By \(f\) we denote the force acting on an individual particle of the suspended phase from the direction of the gas, while \(q\) is the intensity of the inflow of heat to its surface.

Let us express \(f\) in the form of a sum comprising the frictional and Archimedes forces \(f_\mu\) and \(f_A\) and the "associated mass" force \(f_m\) arising from the accelerated motion of the particles relative to the gas \(f = f_\mu + f_A + f_m\). These forces and the heat-transfer intensity \(q\) may be specified most correctly by the fol-
Following relationships:

\begin{align}
\rho_0 = \frac{1}{\omega} \pi d^2 \rho_1^c (u_1 - u_2) |u_1 - u_2|, \\
q = \pi d \lambda_1 (T_1 - T_3)
\end{align}

\begin{align}
\rho_m = \frac{1}{\omega} \pi d^2 \rho_1^c \left( \frac{d u_1}{d t} - \frac{d u_2}{d t} \right), \\
\lambda = \frac{1}{\omega} \pi d^2 \rho_1^c \frac{d u_1}{d t}
\end{align}

where \( T_1 \) is the temperature of the \( i \)-th phase, \( \lambda_1 \) is the thermal conductivity, and \( \rho_1^c \) and \( \lambda_1 \) are the resistance and heat-transfer coefficients, which may be adequately specified subject to various conditions of flow around the particles by the well-known semie empirical relationships for single spheres. In the case of subsonic flows these relationships may be written in the form

\begin{align}
\rho_1^c = 24 / Re + 4 / Re^2, & \quad 0 < Re \leq 700; \\
\rho_1^c = 4.3 (\log Re)^{-1}, & \quad 700 < Re \leq 2000
\end{align}

\begin{align}
Nu = 2 + 0.6 Re / Pr, & \quad Re = \rho_1^c |u_1 - u_2| / \mu_1, \\
Pr = C_v 1 / \rho_1^c
\end{align}

Here \( \mu_1 \) is the dynamic viscosity of the gas, and \( C_v 1 \) is its specific heat at constant pressure.

We may reduce the system (1.1) to the divergent form after first solving the equations of momentum conservation in terms of the derivatives \( d u_1 / d t, d u_2 / d t \). For the mixtures under consideration \( \alpha_2, \rho_1^c / \rho_2^c \ll 1 \); hence, for the sake of simplicity we shall omit terms of order \( \alpha_2, (\rho_1^c / \rho_2^c)^2, \alpha_2 \rho_1^c / \rho_2^c \) from the momentum equations. System (1.1) then takes the following "quasidivergence" form:

\begin{align}
\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 u_1}{\partial x} = 0, \\
\frac{\partial \rho_2}{\partial t} + \frac{\partial \rho_2 u_2}{\partial x} = 0
\end{align}

\begin{align}
\frac{\partial \rho_1}{\partial t} + \frac{\partial \rho_1 u_1}{\partial x} + \frac{3}{2} \frac{\partial p}{\partial x} - \frac{3}{2} \alpha_2 \frac{\partial a}{\partial x} = \frac{3}{2} a_2 \frac{\partial a}{\partial x} + \frac{3}{2} \alpha_2 \frac{\partial a}{\partial x} = \frac{3}{2} a_2 \frac{\partial a}{\partial x}
\end{align}

\begin{align}
(\rho_1 u_1, \rho_1 u_2, \rho_2 u_2, \rho_2 u_2 - \rho_1 u_1) = 0
\end{align}

The mass conservation equations of the phases and also the conservation equations for the total energy of the mixture and the inflow of heat to the particles in (1.1) have a completely divergent form and the momentum conservation equations, an almost divergent form. The incomplete divergence of the momentum conservation equations (terms \( \rho_1^c \partial p / \partial x \)) is due to the allowance made for the Archimedes force and the effects of the "associated masses" in the force interaction between the phases. It should be noted that the corresponding effects of nondivergence are small, since mixtures with low volumetric contents of the suspended phase are under consideration (\( \alpha_2 \ll 1 \)).

We take the equations of state for the phases in the form

\begin{align}
p = p_1^c R T_1, \\
e_1 = c_v 1 T_1, \\
p_1^c = \text{const,} \\
e_2 = c_2 T_2
\end{align}

where \( R \) is the gas constant, \( c_v 1 \) is the specific heat of the gas at constant volume, and \( c_2 \) is the specific heat of the particles. For specified thermodynamic parameters \( \lambda_1, \mu_1, R, c_v 1, c_2 \) the system of equations (1.2)-(1.5) is closed in regions of continuous motion and may be integrated numerically.

The method of through integration used for solving the differential equations (1.4) was developed on the basis of the difference schemes of the "large particle" method [12, 13]. The authors will present the details of the algorithm in a later paper. Here attention will be confined to a few basic principles.

Integration is carried out in two stages. In the first of these the components of the gas suspension are, as it were, "frozen": it is assumed that there is no motion of the phases through the boundaries of the differences or change of heat and momentum between them. From the correspondingly reduced equations of system (1.4), intermediate values of the velocities and total energies of the gas and particles are deduced. The final values of all the parameters of the mixture are calculated in the second stage from the conservation laws relating to individual cells of the difference network. The first and second stages are given specific form after allowing for the particular characteristics of multiphase motion and contain as a constituent part a special algorithm for locating the interfaces. The approximation viscosity of the method with respect to the gas phase