The entry of a wing into the zone of a sharp-edged gust is considered in the linear formulation. The case is studied when the wing velocity is supersonic and its edges satisfy the supersonic flow condition. The gust intensity is considered to be variable, and its edge may move into the undisturbed medium. Equations in finite form are obtained for the forces and moments for a rectangular wing of infinite span, and also for triangular wings with positive and negative sweep, for the case when the gust intensity varies linearly. Sudden envelopment of the wing and penetration of the wing into a gust whose edge is fixed relative to the undisturbed medium are considered.

1. Consider a thin, slightly-cambered wing which moves rectilinearly in an undisturbed medium with a constant supersonic velocity \( u \). We use the wing-fixed rectangular coordinate axes \( Oxyz \) (Fig. 1). Assume that the wing edges satisfy the following condition:

\[
\left| \frac{dY}{dx} \right| > \tan \mu.
\]

Here \( Y(x) \) is the edge equation, \( \mu \) is the half-angle at the apex of the characteristic cone. The normal velocity component at the wing surface is given by the function \( w_0 = w_0(x, y) \).

At the instant of time \( t = 0 \) the wing enters a vertical sharp-edged gust with intensity \( w_1 = w_1(x, y, t) \). The gust edge is a plane normal to the Ox axis, and may move into the undisturbed medium with velocity \( D \) toward the wing. We consider the flow about the wing in the gust to be potential and adiabatic.

For \( t > 0 \) the wing will perform unsteady motions under the influence of the gust and may deform. In this case the variation of the local angles of attack is characterized by the additional normal velocity component \( w_2 = w_2(x, y, t) \) which, generally speaking, is a priori unknown.

The kinematic parameters which characterize the gust intensity, and the additional unsteady motions of the wing relative to the undisturbed medium, will be considered small.

These assumptions make it possible to consider the flowfield about the wing in the linear formulation. The velocity potential \( \varphi(x, y, z, t) \) of the absolute motion of the medium satisfies the equation

\[
(1 - M^2) \varphi_{xx} + \varphi_{yy} + \varphi_{zz} - \frac{1}{c^2} \varphi_{tt} - 2 \frac{\alpha}{c^2} \varphi_{xt} = 0 \tag{1.1}
\]

and the boundary conditions

\[
\varphi |_{t \to 0} = \begin{cases} -w_0(x, y) - w_1(x, y, t) - w_2(x, y, t) & \text{at the wing, (1.2)} \\ 0 & \text{on the shock wave. (1.3)} \end{cases}
\]

Here \( c \) is the speed of sound of the undisturbed medium and \( M \) is the Mach number.

We represent the potential \( \varphi(x, y, z, t) \) in the form

\[
\varphi(x, y, z, t) = \varphi_0(x, y, z) + \varphi_1(x, y, z, t) + \varphi_2(x, y, z, t) \tag{1.4}
\]

where the potential component \( \varphi_0(x, y, z) \) is due to the basic, steady-state motion of the wing with velocity \( u \); the component \( \varphi_1(x, y, z, t) \) is due to the increment of the local angles of attack due to the gust; \( \varphi_2(x, y, z, t) \) is due to the distorted motions and deformation of the wing relative to the undisturbed medium.

Methods of determining \( \varphi_0 \) have been developed, and detailed aerodynamic characteristics are available for wings of various planforms.

The potential \( \varphi_2 \) and the corresponding loading cannot be computed a priori, apart from the consideration of the dynamics of the disturbed motion of the deforming wing.

Let us consider the problem of determining the aerodynamic loading due to the potential \( \varphi_1(x, y, z, t) \). Hereafter we will drop the indexing of \( \varphi \). In this case the potential \( \varphi(x, y, z, t) \) must satisfy Eq. (1.1) and the boundary conditions

\[
[\varphi]_{t \to 0} = \begin{cases} -w_0(x, y) - w_1(x, y, t) & \text{in region 1} \\ 0 & \text{in region 2} \end{cases}
\]

\[
\varphi = 0 \quad \text{on the shock wave. (1.6)}
\]

The pressure \( p(x, y, z, t) \) due to the potential \( \varphi \) is related to it by the expression

\[
p = -\rho_\infty (\varphi + u\varphi_t). \tag{1.7}
\]

Here \( \rho_\infty \) is the density of the undisturbed medium.

The solution of the problem for the potential is contained, as a particular case, in [1]. Satisfaction of the discontinuous boundary conditions therein is achieved by the introduction of a complex region of influence on the wing, whose boundaries depend on time. In this case the integrand in the expression for the potential is considered continuous, including the boundary \( t = 0 \).

For wings of simple planform and simple laws of variation of the gust intensity, it is more convenient to start from the representation
of the potential integrand in terms of generalized functions which have singular properties, but in a simple influence region. In this case, in applications it is often sufficient to have solutions corresponding to the piecewise-linear approximation of the real law of variation of the gust intensity.

2. Let us consider the rectangular wing of infinite span (a) and the triangular wing (sweptback (b) and sweptforward (c)) with supersonic edges (Fig. 2) for the case when the gust intensity varies linearly. We take the solution of Eq. (1.1) in the dimensionless form

\[ \psi(x, y, \tau) = \frac{1}{2\pi} \int_{0}^{\pi} q(x, y, \xi) d\xi , \quad \tau = \frac{x}{\xi} \quad , \quad \xi = \frac{\sin \theta}{\sqrt{1 - \xi^2}} . \] (2.1)

Here \( x, y \) are dimensionless coordinates of a point on the wing; \( \xi, \theta \) are instantaneous dimensionless coordinates; \( b \) is the wing root chord.

The corresponding dimensionless pressure on the wing will be

\[ p(x, y, \tau) = \psi(x, y, \tau) \rho(\tau) \] (2.2)

For the sweptback triangular wing the dimensionless loading at a section along the span is given by the relation

\[ \frac{\partial Z}{\partial x} = \frac{1}{\rho(\tau)} \left[ a_0 \theta_a + a_1 \left( \theta_a - \frac{x}{1 + D} \right) \right] \] (2.3)

Here \( Z \) is the wing lift force; the function \( \Phi \) is introduced following [2].

Let the dimensionless gust intensity be given in the form

\[ w = a_0 + a_1 \delta . \] (2.4)

Here \( s \) is distance in fractions of the chord, \( a_0 \) and \( a_1 \) are dimensionless constants. Then the boundary conditions on the wing will be

\[ |\Psi|_{x=0} = - \left[ a_0 + a_1 \left( \tau - \frac{x}{1 + D} \right) \right] I \left( \tau - \frac{x}{1 + D} \right) . \] (2.5)

Here \( D \) is the dimensionless gust displacement velocity and \( I \) is a unit step function.

We write the equation for the edge of the triangular wing in the form

\[ Y_1, s = \pm x \tan \delta . \] (2.6)

We substitute the boundary conditions (2.5) into

\[ J_0 = I \left( \tau - \frac{x}{1 + D} \right) \left[ \frac{\rho(\tau)}{\Pi} \int_{0}^{\pi} R(\theta) d\theta \right] + \frac{1}{\pi} \int_{0}^{\pi} R(\theta) I \left( \tau - \frac{x}{1 + D} \right) d\theta , \]

for the profile and the sweptforward delta wing

\[ - \int_{0}^{\pi} \rho(\tau) I \left( \tau - \frac{x}{1 + D} \right) d\theta . \] (2.7)

We obtain the total wing lift force by integrating (2.7) and (2.8) in the limits \( 0 \leq x \leq 1 \).

3. Consider, in particular, the cases \( D = 0 \) and \( D = \infty \). We introduce dimensionless loading functions using the formulas

\[ \psi = \frac{Z}{Z_{\infty}} , \quad \Phi = \frac{\rho(\tau)}{\rho(\infty)} , \quad \Theta = \frac{M(\tau)}{M_{\infty}} . \] (3.1)

Here \( Z(\tau), M(\tau) \) are the instantaneous values of the lift force and the pitching moment relative to the \( O_y \) axis, while \( Z_{\infty}, M_{\infty} \) are the steady-state values of these same quantities.

Below we present the formulas for these functions in both cases in finite form. Here the superscripts (1), (2), (3) denote quantities relating to the following intervals, respectively:

\[ 0 < \tau < \frac{M}{M_0} , \quad \frac{M_0}{M} < \tau < \frac{M}{M_0} , \quad \tau > \frac{M}{M_0} . \]

In addition, we denote

\[ w = \frac{\omega_0}{\omega} , \quad B_1 = \frac{1}{\pi} \arccos M \left( 1 - \frac{b^2}{M_0^2} \right) , \]

\[ B_2 = \frac{1}{\pi} \arccos M \left( 1 - \frac{1}{4} \right) . \]