NONSTATIONARY QUASI-ONE-DIMENSIONAL FILTRATION IN AN INHOMOGENEOUS STRATUM

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We consider the quasi-one-dimensional flow of a fluid (gas) in a porous stratum with conductivity and cross section varying along a stream tube. A special form of the differential equation (1.4) for nonstationary quasi-one-dimensional filtration is proposed. We obtain and study some sets of special solutions of this equation including specific solutions of known one-dimensional problems. We establish a relation between the stationary and nonstationary potential and flux distributions in a given filtration region. The laws we find are used as the basis for a method of hydrodynamic probing in inhomogeneous porous media.

1. Since geological information is incomplete, the formulation and solution of problems in underground hydromechanics in complex strata involve extensive diagrammatic presentation of the phenomenon [1]. Hence, in many practically important cases, particularly when the structure of the stratum is unknown in detail, to describe the actual processes it is expedient to use the model of nonstationary quasi-one-dimensional filtration in a stream tube of variable cross section proposed by Charnyi [2] as the basis for a general system of filtration equations. By [2], the equations of continuity and transport in a stream tube D of cross section f and permeability k which are functions of position, f = f(s), k = k(s), can be written as

\[ -\frac{\partial Q}{\partial s} = f \frac{\partial (mp)}{\partial t} , \quad Q = -f k \frac{\partial P}{\partial s} \]  

(1.1)

where Q is the total mass flow rate, P is the formation pressure, \( \rho \) is the density, \( \mu \) is the viscosity of the filtering fluid, \( m \) is the porosity of the stratum at the cross section \( f(s) \), \( s \) is the distance along the streamline in D, which is assumed to be fixed, and \( t \) is the time.

We introduce the independent variable \( \xi \), the Leibenson function \( \Phi \), and the capacity parameter \( \beta \) into Eq. (1.1):

\[ \xi = \xi_0 + \int_{\xi_0}^{\xi} \frac{dl}{f k} \quad (0 < \xi_0 < \xi) , \quad \Phi = \int \frac{\rho dP}{\mu} , \quad \beta = \frac{d(mp)}{d\Phi} \]  

(1.2)

Taking \( \beta \) also to be a function of position, \( \beta = \beta(s) \), we introduce the special variable u:

\[ u = \int \frac{\xi}{\xi_0} \sqrt{\beta k} \, dl \]  

(1.3)

It is easy to see that since \( k > 0 \), \( f > 0 \), the transformation \( s \rightarrow \xi \) is homeomorphic. If we combine (1.1)-(1.3), we obtain an equation for quasi-one-dimensional filtration which, noting the introduced notation, can be written as
The equation for quasi-one-dimensional filtration in its special form (1.4) is the basis for what follows; it is linear in $\phi$ and, replacing $\xi$ by

$$1 + a\xi$$

where $a$ is a real or complex number, does not change its form. It is significant that the variables $\xi$ (1.2) and $u$ (1.3) have an obvious physical sense. The independent variable $\xi$ in (1.4), which behaves as a spatial coordinate, is numerically equal to the potential of a stationary flow in the stream tube under consideration when $\partial \phi / \partial t = 0$, $Q = -1$. From (1.2) and (1.3)

$$\frac{ds}{d\xi} = f k, \quad \frac{du}{d\xi} = f \sqrt{k}$$

Hence, as a result of the functional relationship between $u$, $\xi$, $s$,

$$u = u(s, s_0) = \int_{s_0}^{s} \sqrt{\beta/k} \, dl = \langle \sqrt{\beta/k} \rangle [s - s_0]$$

Obviously the transformation $s \to u$ is also homeomorphic, and $u$ can be considered as a non-dimensional distance

$$\frac{\xi}{\sqrt{k}} \frac{d\ell}{\ell}$$

with factor $\langle \sqrt{\beta} \rangle$.

Using (1.4), we can obtain approximate solutions (within the framework of the quasi-one-dimensional assumptions [2]) of a wide range of nonstationary problems in $D$. In certain important particular cases (1.4) is equivalent to the classical differential equations in filtration and thermal-conductivity theory. This happens, for example, for plane ($c = 0$) and axial ($c = 1$) symmetry of the flows, when for $u = u_c$ and $k, \beta = \text{const}$

$$u_c = b_\xi^0, \quad b, N = \text{const}$$

2. In thermal-conductivity theory we know the solutions of specific problems in two-dimensional ($c = 0$) conduction in specimens with conductivities approximated by power functions in the space coordinate under given constraints on the index of the power [3]. The conductivities of actual strata are not, in general, monotonic functions of the coordinates, and so if we use the above approximation [3] in problems in underground hydromechanics, the qualitative pattern of the phenomenon may be considerably distorted.

A more general description of the inhomogeneities of a quasi-one-dimensional stratum is possible, in which the function (1.3) is approximated, for example, as follows:

$$u = b_\xi^0, \quad b, \alpha = \text{const} > 0$$

since by (1.2), (1.3), (1.6), $u$ is always strictly monotonic and it increases with $\xi$, (2.1) reducing to (1.8) as $\alpha \to 1$ or $\alpha \to \infty$. If $Y(\xi, t)$ is the solution of (1.3) for (2.1), $Y(1 + a\xi, t)$, by (1.5), is the solution of (1.4) for

$$u = b(1 + a\xi)^{\alpha}, \quad b, a, \alpha = \text{const}$$

If we put $a = N/\alpha$ in (2.2), we have

$$u \to u = b e^{\alpha t} \quad \text{for} \quad \alpha \to \infty, \quad N = \text{const}$$

which corresponds to (1.8) when the flow is axially symmetric.

In the space of L-transforms (1.4) takes the form

$$\frac{\partial^2 \theta}{\partial \xi^2} - \left( \frac{du}{d\xi} \right)^2 \lambda \theta = 0$$

(2.4)