Memory Loss Process and Non-Gibbsian Equilibrium Solutions of Master Equations

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The phonon dynamics of a harmonic oscillator coupled to a steady reservoir is studied. In the Markovian limit, the equilibrium is reached through a progressive loss of memory process which involves the moments of the initial distribution. The relationship to the non-Markovian equations of motion and its resolvent poles is settled. As a particular model of the coupling mechanism is adopted, the possibility of non-Gibbsian equilibrium distribution arises, which is analyzed focusing upon the dependence of various parameters of the system on an effective equilibrium temperature.

KEY WORDS: Markovian limit; progressive loss of memory; non-Markovian analysis; non-Gibbsian equilibrium distribution.

1. INTRODUCTION

The features of quantal Brownian motion can be investigated through the quantization of a classical system that generally leads to a quantum Langevin-type equation. The comprehension of the phenomenon may be enhanced if we consider two interacting subsystems neither of which nor the whole system necessarily possesses a priori a classical analog. Regarding this matter, a recent approach to the description of damped collective motion of finite quantal systems such as nuclei has been proposed and various applications have been given. The model involved in these works consists of a harmonic oscillator whose phonons interact with fermions which constitute a heat reservoir; we have recently investigated a more accurate modified version of the original model that takes into account non-Markovian effects in certain configurations at zero...
In the present work we further develop the approach for the general case of nonvanishing temperature.

The quantal Brownian motion of a harmonic oscillator also constitutes a problem of interest in the context of quantum optics. In this framework, the damped oscillator has been generally treated by means of Glauber's $P$ quasiprobability distribution, which, in such a case, moves according to an analytically solvable Fokker–Planck equation. However, the solution through Glauber's $P$ function involves certain disadvantages, since it is unsuitable for describing full quantum configurations, such as:

1. The relaxation of a pure $n$-phonon state, which is the most interesting initial condition when one focuses upon damped collective motion of finite quantal systems such as nuclei. In fact, it is well known that a pure $n$-phonon state cannot be represented by a well-behaved $P$ function (the $P$ representation corresponding to a density matrix $|n\rangle\langle n|$ contains derivatives of delta functions up to order $2n$).

2. The evolution toward a configuration with vanishing equilibrium temperature. In fact, in such a situation it can be shown that both the Fokker–Planck equation and its solutions collapse. This collapsing behavior arises from the fact that all the eigenvectors of the master equation have, at zero temperature, an infinite number of vanishing components and therefore they do not possess a well-behaved associated $P$ function.

The above drawbacks of the $P$ solution led us to explore the direct solutions of the master equation.

This paper is organized as follows: first we solve the master equations for the phonon populations in the Markovian limit (Sections 2 and 3). Since the classical techniques of solution of master equations such as the Kirchhoff or continued-fraction methods are not useful in the present case, we use the characteristic function associated with the probability distribution governed by the master equation. This procedure allows us to find the complete dynamical solution, which is analyzed in Section 3, giving rise to an interesting process with progressive loss of memory. In Section 4 we show that the non-Markovian equations can be easily solved by means of the Markovian solutions. The phonon dynamics in the non-Markovian regime is examined in Section 5 for a particular model of the system-to-reservoir coupling and a non-Gibbsian behavior is observed, which we study, focusing upon the dependence of various parameters of the model on an effective equilibrium temperature. The main results are summarized in Section 6.