Bethe Lattice Spin Glass: The Effects of a Ferromagnetic Bias and External Fields. II. Magnetized Spin-Glass Phase and the de Almeida–Thouless Line

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Received January 16, 1990; final July 12, 1990

In this and the companion paper, we analyze the ± J Ising spin-glass model on the Bethe lattice with fixed uncorrelated boundary conditions. Phase diagrams are derived as a function of temperature vs. concentration of ferromagnetic bonds and, for a symmetric distribution of bonds, external field vs. temperature. In this part we characterize magnetized spin-glass (MSG) phases by divergence of an appropriate susceptibility: at zero field this signals the existence of an intermediate MSG phase; at nonzero field, this is used to identify the de Almeida–Thouless line.

KEY WORDS: Spin glass; Bethe lattice; multicritical point.

1. INTRODUCTION

In this paper we continue our analysis of the ± J Ising spin-glass model on the Bethe lattice with fixed uncorrelated boundary conditions. This analysis began in the companion paper (1) with a discussion of the recursion relation for the distribution of single-site magnetizations ρ(X). Certain aspects of the temperature vs. concentration phase diagram (Fig. 1) were derived rigorously using moment analysis and bifurcation theory. We found that at high temperatures the system is paramagnetic P, i.e., ρ(X) = δ(X). The spin-glass transition (P → SG) corresponds to an instability associated with

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Fig. 1. Phase diagram for the Bethe lattice spin glass, plotted as a function of $p = \tanh(J/kT)$ and the fraction $\lambda$ of ferromagnetic bonds. At high temperatures, the system is paramagnetic. As the temperature decreases, there is a transition to either a spin-glass or a ferromagnetic phase, depending on $\lambda$. Between these phases there is an intermediate magnetized spin-glass (MSG). Like the ferromagnet, the MSG phase has nonzero net magnetization, but it also has glassy susceptibilities. The phase diagram for $\lambda < 1/2$ can be obtained by reflection across the line $\lambda = 1/2$, replacing F and MSG with phases which have long-range antiferromagnetic order. In this paper we determine the phase boundary between the magnetized spin-glass phase and the ferromagnetic phase. All other phase boundaries were determined in ref. 1.

the Edwards–Anderson order parameter $q$, which is the width of the distribution $\rho$. The ferromagnetic transition ($P \rightarrow F$) corresponds to an instability associated with the magnetization $m$, which is the mean of $\rho$. The transition from the spin-glass phase to a magnetized phase, which is this paper we will show is in fact a magnetized spin-glass phase (SG $\rightarrow$ MSG), corresponds to an instability of the symmetric spin glass solution $\rho_{SG}$ to perturbations associated with the mean $m$. However in the neighborhood of the multicritical point, the last phase boundary, $F \rightarrow$ MSG, which corresponds to the instability of the ferromagnetic solution to glassy order, cannot be obtained using the methods employed to determine the other phase boundaries. In Section 2 we demonstrate the existence of this intermediate phase by calculating the Edwards–Anderson susceptibility $\chi_{EA}$, which diverges at the glassy phase boundaries. At non-zero field, divergence of $\chi_{EA}$ also characterizes the de Almeida–Thouless line$^{(2)}$ (Fig. 2), as shown in Section 3.

In Section 4 we conclude this paper with a summary of the results obtained in this and the companion paper, and a discussion of the relationship between the Bethe lattice spin glass and the infinite-range model. We show that in the formal limit where the coordination number of the lattice tends to infinity, the recursion relation becomes the so-called SK