CALCULATION OF THE RATE OF COALESCENCE OF AN EMULSION IN A TURBULENT FLOW

V. M. Entov, V. A. Kaminskii, and E. Ya. Lapiga

The central moment of the theory describing the merging (coalescence) of the drops of an emulsion is determination of the time of the approach of a drop or a number of drops colliding with a given drop in unit time. In the stage immediately preceding the merging of the drops the forces of the hydrodynamic braking of the approaching drops are found to be considerable. The role of these forces has been analyzed earlier for the case of the capture of small drops by large drops in an oncoming flow in the presence of an external electrical field [1] and for the problem of the "Brownian" coalescence of drops, taking account of the effect of the electric double layer and of surface forces of interaction [2-4]. The present article considers the approach of drops with turbulent diffusion in an electrical field. Of the greatest interest is the sharp slowing of the approach due to the hydrodynamic interaction of the drops, considerably sharper than in the case of molecular diffusion [2]. As a result, the sharp acceleration of the approach and coalescence of drops with the action of an electrical field on an emulsion in a turbulent flow becomes understandable.

1. Effective Diffusion Coefficient for a Particle under the Action of a Random Force

The diffusion coefficient D for a particle moving along the x axis under the action of a random force F and encountering a resistance can be obtained by averaging of the Langevin equation [5]

\[-H\dot{x} + F = 0\]  

where H is the resistance coefficient, and is found equal to

\[D = BH^{-1}, \quad B = \langle f(t)^2 \rangle / 2t, \quad f(t) = \int F(t) dt\]  

In the presence of thermodynamic equilibrium we have the classical relationships

\[B = HkT, \quad D = kTH^{-1}\]  

In the case where the resistance coefficient varies at distances larger than the displacement of a particle by one pulsation of the force, in relationship (1.3) H can be replaced by H(x). The presence of regular forces (nonrandom), whose scale is large in comparison with the scale of the pulsations, does not change the form of the diffusion coefficient.

Equation (1.1), in an inertialess approximation, describes the motion of a particle in a viscous liquid under the action of a random force, independently of the validity of the first relationship of (1.3), following from the condition of static equilibrium (for example, if the force F is not connected with the thermal fluctuations). If this relationship is not satisfied, the inverse proportionality of the diffusion coefficient to the resistance H breaks down. Thus, if it is assumed that the random force does not depend on H (for example, there is an external given random field), then the diffusion coefficient is found to be inversely proportional to \(H^2\). If the random force at any moment of time is proportional to H, then the diffusion coefficient is found to be independent of the resistance.

The latter situation is realized with the diffusion of a free particle in a field of turbulent pulsations on a scale less than the internal scale of the turbulence [6]. Let us consider this case in more detail. The internal scale of the turbulence is distinguished by the fact that it corresponds to a "pulsational" Reynolds number equal to unity, i.e., all the relative displacements inside an isolated element of the liquid are determined by the Stokes equations of inertialess motion. In this approximation, the velocity of the suspended particles \( \mathbf{r} \) coincides with the local velocity of the liquid \( \mathbf{v} \), which is a random function, determined by the parameters of the external turbulent flow, i.e., \( \mathbf{r} = \mathbf{v} \).

Let us now consider the approach of a spherical particle of radius \( a \) to a wall, around which there is turbulent flow. If the particle is sufficiently small, then the principal stage of the approach of the particle to the wall is the motion of the particle in a viscous sublayer, which permits writing the equation of motion in an inertialess approximation (we limit ourselves to a consideration of motion along the \( x \) axis, perpendicular to the wall):

\[
F_g + F_r = 0, \quad F_g = \eta \mathbf{v}, \quad F_r = -\mu \mathbf{u} \tag{1.4}
\]

Here \( F_g \) is the force which would act on a fixed particle from the side of the flow around it; \( F_r \) is the force of the resistance to the inherent motion of a particle; \( \mu \) is the velocity of a particle.

At large distances from the wall, the particle moves together with the liquid:

\[
H_g = H_r = H_e = 6\pi \mu a, \quad v = u \tag{1.5}
\]

With approach to the wall, both coefficients \( H_g \) and \( H_r \) vary. However, the coefficient \( H_g \) varies comparatively little (this follows, for example, from [7]), while the coefficient \( H_r \) rises sharply and reverts to infinity at the moment when the particle touches the wall.

In accordance with (1.4), the random motion of a particle is determined by the relationship

\[
u - \dot{\mathbf{x}} = H_g \mathbf{v} / H_r \tag{1.6}
\]

Obviously, on a small scale (so long as the dependence of \( H_g \) and \( H_r \) on the coordinate \( x \) can be neglected), Eq. (1.6) describes a motion similar to the motion of an unconstrained particle (relationship (1.5) with the similarity coefficient \( H_g / H_r \)). Therefore, from (1.5) it follows that the effective diffusion coefficient of a particle in a field of turbulent pulsations near the wall is equal to

\[
D(x) = D_0(x) \left( H_g / H_r \right)^{-1}, \quad D_0 = \left( \left( \frac{\langle \int u(\tau) d\tau \rangle}{\tau} \right) \right) / 2t \tag{1.7}
\]

Here \( D_0 \) is the diffusion coefficient with the unconstrained motion of a particle.

Thus, in the case of turbulent diffusion the diffusion coefficient is inversely proportional to the square of the coefficient of the resistance to motion of the particle [but not to its first power, as in the case of molecular diffusion, i.e., formula (1.3)].

In relationship (1.7) the diffusion coefficient of unconstrained particles is assumed to depend explicitly on the distance to the wall, which reflects the decrease in the coefficient of turbulent diffusion with an approach to the wall due to damping of pulsations in the viscous sublayer.

In accordance with existing data

\[
D_0(x) = v(x/\delta)^\alpha \tag{1.8}
\]

Here \( \delta \) is the thickness of the viscous sublayer; \( v \) is the kinematic viscosity of the liquid. Formula (1.8) takes account of the turbulent transfer of an impurity and of the momentum. With respect to the value of the exponent \( \alpha \), up to the present time there is no complete clarity; it is generally assumed that \( \alpha = 3-4 \) [6, 8, 9]. In what follows, no account is taken of the change in the coefficient \( H_g \) with distance from the wall, we set \( H_g = H_0 \), and for the coefficient \( H_r \) we adopt the approximation

\[
H_r = H_r \left[ 1 + \alpha (x-a)^{-1} \right] = H_r \left( x-a \right)^{-1} \tag{1.9}
\]

correctly reflecting the asymptotic curve of \( H_r \) with \( x \rightarrow a \) and \( x \rightarrow \infty \).

From (1.7)-(1.9), we find

\[
D(x) = v(x/\delta)^\alpha \left[ (x-a)/x \right]^2 \tag{1.10}
\]