HYDRODYNAMICS OF UNDERGROUND GAS STORAGE VAULTS

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The problem of determining the change of the volume and pressure of a gas cavity with time in a mildly sloping seam of the dome type, used as an underground gas storage vault, is solved by the method of combined asymptotic expansions; the gas storage vault has an arbitrary law of variation of discharge with time. A comparison is carried out with the existing exact solution for the case of a seam with constant thickness; good agreement is revealed. Calculations are carried out on the dynamics of a gas cavity in an actual seam. The case of a seam with a steep dome and a strong anisotropic permeability is also considered.

§ 1. We shall consider the problem of a gas with a discharge Q(t) which is varying arbitrarily with time in a water-bearing seam having a domed section [Fig. 1, 1) gas; 2) water]. The seam is assumed to be infinite and has a constant thickness h remote from the dome. The pressure in the gas cavity p(t), which is assumed to be identical over the whole volume at every instant, and the volume V(t) occupied by the gas will be found.

In the aqueous zone the pressure satisfies the equation for the filtration of an elastic liquid in a compressed porous medium which for the case of axisymmetrical flow being considered has the form

\[ \frac{\partial \bar{p}}{\partial t} = \kappa \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{p}}{\partial r} \right) + \frac{\partial \bar{p}}{\partial z} \right] \]  

Here \( \kappa \) is the piezoconductivity of the seam.

This equation must be solved for the following conditions. At the gas-water interface, which is assumed to be horizontal, the gas and water pressures are identical:

\[ p = p_w, \quad z = z_i(t) \]  

The rate of sinking of the interface is equal to the actual average speed of motion of water particles through the porous medium at the interface,

\[ m \sigma^*_a \frac{dz}{dt} = -\frac{k}{\mu} \left( \frac{\partial \bar{p}}{\partial z} - \rho_0 g \right), \quad z = z_i \]  

Here \( m \) is the coefficient of porosity, \( \sigma^*_a \) is the average value of the gas saturability, taking account of the incomplete displacement of water by the gas, \( k \) is the permeability factor, and \( \rho_0 \) and \( \mu \) are the density and viscosity of water.

At the initial instant of time

\[ p + \rho_0 g z = p_i = \text{const}, \quad t = 0 \]  

At infinitely remote points at any instant

\[ p + \rho_0 g z = p_i = \text{const}, \quad r \to \infty \]  

In addition, the condition which expresses the equivalence of the pumped mass of gas and the reserve of gas in the seam must be fulfilled:

\[ \rho_0 \int_0^t Q(t) \, dt = V(t) \rho_0, \quad V(t) = m \sigma^*_a \int_0^r r^2 z'(r) \, dr \]  

where $p_0$ is the normal atmospheric pressure, $Q(t)$ is the volume flow rate of the gas, reduced to normal conditions, $V(t)$ is the volume of the porous space occupied by the gas, and $z = z(r)$ is the equation for the surface area of the dome.

Equation (1.6) can be written in the form

$$
\varepsilon \int_0^t \frac{Q(t)}{\left|Q\right|_{\text{max}}} \, dt = \frac{P_0}{4\rho_p c h} \int_0^r r^2 z'(r) \, dr,
$$

$$
\varepsilon = \frac{\left|Q\right|_{\text{max}} P_0}{4\pi m \sigma \varepsilon h p_c}
$$

(1.7)

Here $\left|Q\right|_{\text{max}}$ is the modulus of the maximum value of the gas flow rate.

In the future, it will be assumed in solving the problem that the parameter $\varepsilon$ takes on small values.

The condition $\varepsilon \ll 1$ is fulfilled in many cases of practical importance, for example, when $Q_{\text{max}} = 10^5 \text{ m}^3/\text{h}$, $p_0 = 1 \text{ atm}$, $p_c = 100 \text{ atm}$, $m = 0.2$, $\sigma = 0.5$, $\sigma = 10^4 \text{ cm}^2/\text{sec}$, and $h = 10 \text{ m}$, we obtain $\varepsilon \approx 10^{-3}$. Acceptance of this condition permits the asymptotic method [1, 2] to be used for solving the problem, and this simplifies the solution considerably.

§ 2. We shall reduce Eq. (1.1) and the conditions at the boundaries to a dimensionless form with some special choice of the dimensionless variables. We denote

$$
P = \frac{p}{p_c}, \quad \xi = \frac{z}{H}, \quad \rho = \frac{r}{R}, \quad \tau = \frac{\varepsilon t}{R^2}, \quad p* = \frac{p}{p_c}
$$

(2.1)

where $H$ and $R$ are the height and radius of the dome.

Relations (1.1)-(1.5) and (1.7) assume the form

$$
\varepsilon \frac{\partial P}{\partial \tau} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial P}{\partial \rho} \right) + \frac{R_i^2 \partial^3 P}{H^3 \partial z^3} = 0
$$

(2.2)

$$
P = p*, \quad \xi = \frac{z}{H}, \quad \rho = \frac{r}{R}, \quad \tau = \frac{\varepsilon t}{R^2}, \quad p* = \frac{p}{p_c}
$$

(2.3)

$$
\varepsilon \frac{\partial^2 \xi}{\partial \tau^2} = -4\pi \rho \frac{R_i^2}{H^2} \left( \frac{\partial P}{\partial \xi} + \frac{p_c \rho g H}{p_c} \right), \quad \beta = \frac{k p_c}{4\pi m \sigma \varepsilon h}
$$

(2.4)

$$
P + \frac{p_c \rho g H}{p_c} \xi = 1, \quad \tau = 0
$$

(2.5)

$$
P + \frac{p_c \rho g H}{p_c} \xi = 1, \quad \rho = \infty
$$

(2.6)

$$
\int_0^t \frac{Q(t)}{\left|Q\right|_{\text{max}}} \, dt = \frac{p_c \rho g H}{4\rho_p c h} \int_0^\xi \rho^2 \xi^2 (\rho) \, d\rho
$$

(2.7)

The condition of smallness of the parameter $\varepsilon$ facilitates the solution of the problem, but it is not possible simply to neglect the term with $\varepsilon$ in Eq. (2.2), since the solution of the equation "shortened" in this way (Laplace's equation) at high values of $\rho$ behaves as $\ln \rho$ and does not satisfy the condition of Eq. (2.6) at infinity. Thus, the classical situation arises for the use of the method of singular perturbations since Eq. (2.2) can be solved for $\varepsilon = 0$ in the vicinity of the gas cavity, but remote from it the necessity arises of finding another solution of Eq. (2.2) which satisfies the condition of Eq. (2.6) at infinity. We shall find both solutions and we shall suppose, in accordance with the general scheme of the method of singular perturbations, that there exists a certain range of values of $\rho$ in which they are both applicable. In order to find the external expansion of the function $P$, valid in the vicinity of the dome, we put $\varepsilon = 0$ in Eq. (2.2). In this case we arrive at Laplace's equation

$$
\frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial P}{\partial \rho} \right) + \frac{R_i^2 \partial^2 P}{H^2 \partial z^2} = 0
$$

(2.8)