Combining solutions (4.9) and (2.19), we find the expression for the gas pressure.

We write it in dimensionless form through the dimensionless time \(x = t/T_0\):

\[
p^* = 1 + \frac{q(x)}{4\pi \ln \frac{2.25\sqrt{x}}{R_0}} - \frac{4}{4\pi} \int x-x_i \frac{q(x) - q(x_i)}{x-x_i} dx_i - \left[ \frac{\rho g H}{p_c} + \frac{H k_c}{\pi R_0^2 k_i} q(x) \right] \frac{x_i}{2} + \frac{2 H k_c}{\pi R_0^2 k_i} q(x) \frac{x_i}{2} \tag{4.10}
\]

The systems (4.10), (3.10), and (3.11) permit the problem to be solved.

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LITERATURE CITED


TWO-PHASE FILTRATION TO AN IMPERFECT BOREHOLE IN AN INHOMOGENEOUS PRODUCING STRATUM

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A finite-difference method is proposed for solving the filtration equations of a two-phase liquid passing into a borehole partly revealing an oil stratum. The results of a finite-difference calculation of the dynamics of borehole flooding are compared with the solutions obtained by the zonal-linearization method, as proposed by Abramov, Danilov, and Kats [1].

It is well known that in the case of the filtration of liquids of different weights steady-state alluvial fans may sometimes be formed. Many authors [2-4] after making certain assumptions regarding the potential distribution in the oil-saturated part of a stratum have determined the maximum possible water-free discharge corresponding to a steady-state fan. In the dynamic approach the steady-state fan is sought as the limit to a series of intermediate positions. In one earlier paper [5] the interface between the oil and the water was followed until it arrived at a steady-state position, and the limiting discharges for various degrees of revelation were then determined.

We shall employ the dynamic approach in this paper as well; however, we shall here determine the steady-state fan as the limiting state of a two-phase fan, achieved by lifting the fan and at the same time segregating the liquids.

Using a difference method we shall calculate the dynamics of fan formation, determine the limiting water-free discharges, and compare these with the discharges based on the Masket-Charnyi theory of the steady-state fan [4].

§ 1. Let us consider a semiinfinite stratum with an impermeable roof in which the initial oil-saturated layer of thickness \(H\) is revealed to a depth \(b\) by a vertical linear sink - a borehole with a volumetric delivery \(Q\). The water-saturated thickness of the stratum is infinite [Fig. 1, 1) water; 2) oil]. The liquids are assumed to be incompressible and immiscible, the stratum is inhomogeneous and anisotropic, but the capillary effect is not taken into account.

The equations governing the simultaneous filtration of the water and oil may be written in the form

$$
\begin{align*}
\omega_r &= -\left( \frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} \right) k_r \frac{\partial p}{\partial r} \quad \text{(1.1)} \\
\omega_z &= -\left( \frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} \right) k_z \frac{\partial p}{\partial z} + \frac{f_1}{\mu_1} k_t = 0 \quad \text{(1.2)} \\
\omega_{iw} &= \omega_{iq} + \Delta \gamma_i \phi_i \frac{f_1}{\mu_1}, \quad \phi = f_1 / (f_1 + \mu_1 f_1) \quad \text{(1.3)} \\
\frac{1}{r} \frac{\partial}{\partial r} \left( r \omega_r \right) + \frac{\partial \omega_z}{\partial z} + m \frac{\partial \sigma}{\partial t} &= 0 \quad \text{(1.4)}
\end{align*}
$$

Here $k_r(r, z), k_z(r, z)$ are the components of the absolute permeability of the medium along the $r$ and $z$ coordinate axes; $m$ is the porosity; $p$ is the pressure; $\sigma$ is the water saturation; $f_i$ is the relative phase permeability for the $i$-th phase; $\Delta \gamma = \gamma_1 - \gamma_2$, $\gamma_1 = \rho g$; $g$ is the gravitational acceleration; $\rho_1$ and $\mu_1$ are the density and viscosity of the $i$-th phase; $\omega_{iw}, \omega_{iq}$ are the total velocity of the liquid and the velocity of the first (aqueous) phase; the indices $r$ and $z$ define the corresponding velocity components; $t$ is the time.

For a homogeneous-anisotropic stratum with an anisotropy factor of $\lambda^2 = k_r/k_z$ a change of variables

$$
\xi = \frac{r}{k_H}, \quad \eta = \frac{z}{H}, \quad \tau = \frac{3Q t}{2m \lambda^2 H^2}, \quad \Phi = \frac{2\pi \sigma H}{3 \mu Q} (p - \gamma z)
$$

transforms Eqs. (1.1)-(1.4) to the form

$$
\frac{\partial \sigma}{\partial \tau} - \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{f_1}{\mu_1} \frac{\partial \Phi}{\partial \xi} \right) - \frac{\partial}{\partial \eta} \left[ f_1 \left( \frac{\partial \Phi}{\partial \eta} - G_i \right) \right] = 0
\quad \sigma_r + \sigma_z = 1 \quad \text{(1.6)}
$$

where $G_i = \frac{2\pi k_i H^2 \Delta \gamma_i}{3Q \mu_i}$, $\mu_0 = \mu / \mu_n$, $\Delta \gamma = \gamma_1 - \gamma_2$.

Here the index $i = 1$ refers to the water and $i = 2$ to the oil. The condition of the impermeability of the roof and the boundary condition at the operating borehole before the breakthrough of the water take the form

$$
\frac{f_1}{\mu_1} \left( \frac{\partial \Phi}{\partial \eta} - G_i \right) = 0 \quad (\eta = 0) \quad \text{(1.7)}
$$

$$
\lim_{\xi \to \infty} \frac{\partial \Phi}{\partial \xi} = -\frac{1}{3\beta} \quad (\eta < \beta) \quad \text{(1.8)}
$$

Here $\beta = b/H$ is the degree of revelation (opening) of the stratum by the borehole. After the breakthrough of water into the borehole, the total delivery $Q$ is distributed along the length of the borehole in such a way that the flow of liquid through unit length is proportional to its mobility.

The initial conditions

$$
\sigma(\xi, \eta, 0) = \begin{cases} 
0, & \eta \leq 1 \\
1, & \eta > 1 
\end{cases} \quad \text{(1.9)}
$$

correspond to the limiting values on the relative phase permeability curves.