Combining solutions (4.9) and (2.19), we find the expression for the gas pressure. We write it in dimensionless form through the dimensionless time \( x = t/T_0 \):

\[
p^* = 1 + \frac{q(x)}{4\pi \ln \frac{2.25x}{R}} - \frac{1}{4\pi} \int_0^\infty \frac{2(q(x) - q(x_i))}{x-x_i} dx_i - \left[ \frac{\rho_s g H}{p_s} + \frac{H k_t}{\alpha R_s^2 k_i} q(x) \right]_{x=x_i}^0 + \frac{H k_t}{\alpha R_s^2 k_i} q(x) \frac{t_s}{2} \tag{4.10}
\]

The systems (4.10), (3.10), and (3.11) permit the problem to be solved. The authors thank V. D. Epishin for the calculations performed by him.

**LITERATURE CITED**


**TWO-PHASE FILTRATION TO AN IMPERFECT BOREHOLE IN AN INHOMOGENEOUS PRODUCING STRATUM**

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A finite-difference method is proposed for solving the filtration equations of a two-phase liquid passing into a borehole partly revealing an oil stratum. The results of a finite-difference calculation of the dynamics of borehole flooding are compared with the solutions obtained by the zonal-linearization method, as proposed by Abramov, Danilov, and Kats [1].

It is well known that in the case of the filtration of liquids of different weights steady-state alluvial fans may sometimes be formed. Many authors [2-4] after making certain assumptions regarding the potential distribution in the oil-saturated part of a stratum have determined the maximum possible water-free discharge corresponding to a steady-state fan. In the dynamic approach the steady-state fan is sought as the limit to a series of intermediate positions. In one earlier paper [5] the interface between the oil and the water was followed until it arrived at a steady-state position, and the limiting discharges for various degrees of revelation were then determined.

We shall employ the dynamic approach in this paper as well; however, we shall here determine the steady-state fan as the limiting state of a two-phase fan, achieved by lifting the fan and at the same time segregating the liquids.

Using a difference method we shall calculate the dynamics of fan formation, determine the limiting water-free discharges, and compare these with the discharges based on the Masket—Charnyi theory of the steady-state fan [4].

§ 1. Let us consider a semi-infinite stratum with an impermeable roof in which the initial oil-saturated layer of thickness \( H \) is revealed to a depth \( b \) by a vertical linear sink—a borehole with a volumetric delivery \( Q \). The water-saturated thickness of the stratum is infinite [Fig. 1, 1) water; 2) oil]. The liquids are assumed to be incompressible and immiscible, the stratum is inhomogeneous and anisotropic, but the capillary effect is not taken into account.

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The equations governing the simultaneous filtration of the water and oil may be written in the form

\[ w_r = - \left( \frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} \right) k_r \frac{\partial p}{\partial r} \]  
\[ w_z = - \left( \frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} \right) k_z \frac{\partial p}{\partial z} + \left( \frac{f_1 \gamma_1}{\mu_1} + \frac{f_2 \gamma_2}{\mu_2} \right) k_z \]  
\[ \frac{1}{r} \frac{\partial}{\partial r} (rw_r) + \frac{\partial w_z}{\partial z} = 0 \]  
\[ w_{ir} = w_1 q \]  
\[ w_{iz} = w_1 q + \Delta \gamma k_r \frac{f_1}{\mu_1} \frac{\partial q}{\partial r} \]  
\[ \frac{1}{r} \frac{\partial}{\partial r} (rw_{ir}) + \frac{\partial w_{iz}}{\partial z} + m \frac{\partial \sigma}{\partial t} = 0 \]  

Here \( k_r(r, z), k_z(r, z) \) are the components of the absolute permeability of the medium along the \( r \) and \( z \) coordinate axes; \( m \) is the porosity; \( p \) is the pressure; \( \sigma \) is the water saturation; \( f_i \) is the relative phase permeability for the \( i \)-th phase; \( \Delta \gamma = \gamma_1 - \gamma_2 \); \( \gamma_i = \rho_i g \); \( g \) is the gravitational acceleration; \( \rho_1 \) and \( \mu_1 \) are the density and viscosity of the \( i \)-th phase; \( \mu_0 = \frac{\mu_1}{\mu_2} \), \( w_r \), \( w_z \) are the total velocity of the liquid and the velocity of the first (aqueous) phase; the indices \( r \) and \( z \) define the corresponding velocity components; \( t \) is the time.

For a homogeneous-anisotropic stratum with an anisotropy factor of \( \lambda^2 = \frac{k_r}{k_z} \) a change of variables

\[ \xi = \frac{r}{k_H}, \quad \eta = \frac{z}{H}, \quad \tau = \frac{3Q t}{\lambda^2 H^2}, \quad \Phi = \frac{2\pi k_H H^2}{3Q} (p - \rho g z) \]  

transforms Eqs. (1.1)-(1.4) to the form

\[ \frac{\partial \sigma_1}{\partial \tau} - \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \xi \frac{f_1}{\mu_1} \frac{\partial \Phi}{\partial \xi} \right) - \frac{\partial}{\partial \eta} \left[ \frac{f_1}{\mu_1} \left( \frac{\partial \Phi}{\partial \eta} - G_i \right) \right] = 0 \]  
\[ \sigma_1 + \sigma_2 = 1 \]  
\[ G_i = \frac{2\pi \lambda H \Delta \gamma_i}{3Q \mu_2}, \quad \mu_0 = \mu_1/\mu_2, \quad \Delta \gamma_i = \gamma_i - \gamma_2 \]  

Here the index \( i = 1 \) refers to the water and \( i = 2 \) to the oil. The condition of the impermeability of the roof and the boundary condition at the operating borehole before the breakthrough of the water take the form

\[ f_1/m \left( \frac{\partial \Phi}{\partial \eta} - G_i \right) = 0 \]  
\[ \lim_{\xi \to 0} \frac{\partial \Phi}{\partial \xi} = -\frac{1}{3\beta} \]  
\[ \eta_\beta \]  

Here \( \beta = b/H \) is the degree of revelation (opening) of the stratum by the borehole. After the breakthrough of water into the borehole, the total delivery \( Q \) is distributed along the length of the borehole in such a way that the flow of liquid through unit length is proportional to its mobility.

The initial conditions

\[ \sigma(\xi, \eta, 0) = \begin{cases} 0, & \eta \leq 1 \\ 1, & \eta > 1 \end{cases} \]  

correspond to the limiting values on the relative phase permeability curves.