DEVELOPMENT OF TWO- AND THREE-DIMENSIONAL PERTURBATIONS IN THE CASE OF AN IONIZATION INSTABILITY IN A CHANNEL WITH NONCONDUCTING WALLS

V. M. Zubtsov and O. A. Sinkevich

An ionization instability of a plasma bounded by nonconducting walls is investigated taking into account electron thermal conduction. The wave vector is considered directed at some angle to the magnetic field direction. Perturbations with a wave vector orthogonal to the magnetic field induction vector turn out to be most unstable. A relatively simple formula to compute the neutral curve separating the stability and instability domains is obtained.

§1. A nonequilibrium magnetized plasma is considered with the initial equation being the Boltzmann equation for an electron distribution function. Let us discuss the system of equations describing the behavior of electrons in a continuous medium approximation. The zero and second moments of the Boltzmann equation yield the following equations for the change in the number of electrons and for their energy:

\[
\frac{dn_e}{dt} + \text{div} q_n = N_+ - N_-
\]  

\[
I \left( \frac{\partial n_e}{\partial t} + \text{div} q_n \right) + \frac{3}{2} \frac{\partial n_e T_e}{\partial t} + \text{div} q_T = jE - F_-
\]

Here \(q_n, q_T\) are the electron and energy fluxes, respectively, \(n_e\) is the electron concentration, \(T_e\) is the electron temperature, \(j\) is the electrical current density, \(E\) is the electrical field intensity, \(N_+\) is a term taking account of generation of electrons and dependent on the kind of ionization process, \(N_-\) depends on the kind of recombination process, \(F_-\) takes account of the energy transmitted from electrons to heavy particles during elastic collisions, and \(I\) is the ionization potential.

We find the following dependences of the particle and energy fluxes on the electrical field, and the concentration and temperature gradients from the solution of the Boltzmann equation for the case of heavy particles at rest (or in a coordinate system moving with the velocity of the heavy particles) presented in [1]:

\[
q_n + q_n \times \Omega = -n_e \left\{ \mu_e E + D_e \left[ \frac{\nabla n_e}{n_e} + \left( \frac{3}{2} \right) \frac{V T_e}{T_e} \right] \right\}
\]  

\[
q_T + q_T \times \Omega = -\lambda_e \nabla T_e + T_e^g (q_n + q_n \times \Omega)
\]

where \(\mu_e \) is the electron mobility; \(D_e, \lambda_e\) are the electron diffusion and thermal conductivity coefficients, respectively; \(\Omega\) is the Hall parameter; and \(\xi\) is a numerical coefficient determined by the dependence of the electron mean free path on the velocity.

Since the electrical field density is determined just by the electron flux in a coordinate system coupled to the velocity of the heavy particles, \(1.3\) is equivalent to Ohm's law:

\[
j + j \times \Omega = \sigma \left\{ E + \frac{D_e}{\mu_e} \left[ \frac{\nabla n_e}{n_e} + \left( \frac{3}{2} \right) \frac{V T_e}{T_e} \right] \right\}
\]

Here \(\sigma\) is the coefficient of electrical conductivity.

Using \(1.4, 1.5\), and the Einstein relationship \(D_e/\mu_e = T_e/\epsilon_e\), let us convert the energy equation \(1.2\) into

If the magnetic field is directed along the z axis, then the components of the tensor \( M^+ \) are \( a_{11} = a_{22} = 1, a_{13} = a_{23} = a_{31} = 0, -a_{12} = a_{21} = \Omega, a_{33} = 1 + \Omega^2 \).

§ 2. Let us consider that ionization equilibrium holds. In this case, (1.1), (1.5), (1.6), and the Maxwell equation written under the assumption that the Reynolds magnetic number is small yield the following system of equations:

\[
\begin{align*}
\text{rot } E &= 0, \quad \text{div } j = 0, \quad n_0 = n_e(T_e), \\
\left[I + \frac{3}{2} T_e \left(1 + \frac{3}{2} \frac{\partial \ln T_e}{\partial n_e}\right)\right] \frac{\partial n_e}{\partial t} + \frac{T_e}{e} \left(1 - \frac{3}{2} \frac{\partial \ln T_e}{\partial n_e}\right) \nabla \ln n_e - 3 \frac{\lambda_e}{1 + \Omega^2} \nabla T_e &= \frac{j^2}{\sigma} - \frac{3}{2} \left(T_r - T_e\right) \delta n_e V 
\end{align*}
\]

(2.1)

Here \( T_a \) is the temperature of the heavy particles, \( \delta \) is the energy fraction transmitted by an electron during collisions with heavy particles, and \( \nu \) is the collision frequency between electrons and heavy particles.

Let us investigate the stability of a plasma described by the system of equations (2.1) in a domain bounded by nonconducting walls along the x axis between which the spacing is b. A homogeneous magnetic field is directed along the z axis and the unperturbed current, along the y axis.

Introducing the potential \( E^x = \text{grad } \varphi \), we obtain after linearizing Ohm's law and eliminating the current perturbations by using the charge conservation law

\[
\frac{\partial}{\partial x} \left( \frac{\partial^2 \varphi'}{\partial x^2} + \frac{\partial^2 \varphi'}{\partial y^2} + \left(1 + \Omega^2\right) \frac{\partial^2 \varphi'}{\partial z^2} \right) - i_{e0} \frac{\partial \Omega_e}{\partial n_e} \left( \frac{\partial \varphi}{\partial x} + \Omega_e \frac{\partial \varphi}{\partial y} \right) n_e' + \frac{i_{e0}}{\sigma} \left(1 + \Omega^2\right) \frac{\partial n_e}{\partial n_e} \frac{\partial n_e}{\partial y} = 0
\]

(2.2)

The subscript 0 denotes the unperturbed quantities and the prime denotes their perturbations.

The electron temperature in a stationary homogeneous state is determined by the balance between the Joule heat and the energy losses during elastic collisions:

\[
\frac{i_{e0}^2}{\alpha_e(T_e)} = \frac{3}{2} \delta T_e n_e(T_e) \nu_0(T_e)
\]

Taking account of this relationship, the linearized energy equation is

\[
\begin{align*}
\left[I + \frac{3}{2} T_e \left(1 + \alpha_e\right)\right] \frac{\partial n_e}{\partial t} + \frac{T_e}{e} \left(1 - \frac{3}{2} \alpha_e\right) \frac{\partial n_e}{\partial y} - \\
- \frac{T_{e0} \alpha_e}{\nu_0 \left(1 + \Omega^2\right)} \left[\left(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) + \left(1 + \Omega^2\right) \frac{\partial^2}{\partial z^2}\right] n_e' = \\
- \frac{i_{e0}^2}{\sigma_e} \left\{\left(1 - \alpha_e\right) \left(1 - \frac{T_e}{T_0}\right) + \alpha_e + \alpha_e \right\} n_e' \quad n_0 - 2 \frac{i_{e0}}{\sigma_e}
\end{align*}
\]

(2.3)

We obtain from Ohm's law

\[
\frac{i_{e0}'}{i_{e0}} = -\frac{\Omega_e}{1 + \Omega_e^2} \frac{\partial n_e}{\partial y} + \frac{1}{1 + \Omega_e^2} \frac{i_{e0}}{\sigma_e} \left(\Omega_e \frac{\partial \varphi}{\partial x} - \frac{\partial \varphi}{\partial y}\right)
\]

(2.4)

Let us select the characteristic quantities and let us introduce the dimensionless parameters

\[
l_s = b, \quad l_r = \left[I + \frac{3}{2} T_e \left(1 + \alpha_e\right)\right] n_0 \sigma_0 j_{e0}, \quad \delta = \frac{n_e'}{n_0}, \quad \Phi = \frac{\sigma_0}{j_{e0} b \Omega_e^{\alpha_e} \left(1 + \alpha_e\right)}
\]

(2.5)

Using these definitions, let us write (2.2) and (2.3) [but first substituting (2.4) into (2.3)] in the dimensionless form

\[
\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} + \left(1 + \Omega_e^2\right) \frac{\partial^2 \varphi}{\partial z^2} - \alpha_e \Omega_e \left(\frac{\partial \varphi}{\partial x} + \Omega_e \frac{\partial \varphi}{\partial y}\right) + \left(1 + \Omega_e^2\right) \alpha_e \frac{\partial \varphi}{\partial y} = 0
\]

(2.6)