EHD EQUATIONS AND TRANSPORT COEFFICIENTS IN A STRONG ELECTRIC FIELD

V. V. Gogosov, V. A. Polyanski, I. P. Semenova, and A. E. Yakubenko

Izv. AN SSSR. Mekhanika Zhidkosti i Gaza, Vol. 4, No. 2, pp. 31-45, 1969

The methods of the kinetics theory are used to obtain a closed system of equations describing the behavior of a multicomponent, partially ionized gaseous mixture in an electromagnetic field in which the space charge is significant. A criterion is presented that makes it possible to separate the problem of finding the magnetic field from that of finding the other defining parameters. Expressions are obtained for the viscosity stress tensors and the thermal and diffusive fluxes; the transport coefficients are calculated in the strong electric field. The relations for the friction force and the energy exchange between components during particle collisions are analyzed. The equations for a mixture consisting of neutral particles and charged particles of a single sign are discussed in detail. The dimensionless EHD criteria are written and analyzed. Possible simplifications of the system of equations are examined, and various forms of Ohm's law are discussed. Weak discontinuities in EHD are analyzed. The equations of EHD under various assumptions have also been considered in several studies* and in [1-5].

1. Equations of motion of a multicomponent plasma with a space charge in an electric field. We consider a multicomponent, partially ionized plasma in an electromagnetic field. We take the following quantities as the parameters characterizing the macroscopic state of each component of the plasma: density $\rho_a$, pressure $P_a$, velocity $u_a$, internal energy per unit mass $e_a$, and temperature $T_a$. We also introduce parameters that define the state of the medium as a whole: space charge, total density and pressure, average mass velocity, internal energy per unit mass, and temperature of the mixture:

$$ q = \sum_a e_a n_a, \quad \rho = \sum_a \rho_a = \sum_a m_a n_a, \quad \rho = \sum_a \rho_a, $$

$$ u = \frac{1}{\rho} \sum_a \rho_a u_a, \quad \varepsilon = \frac{1}{\rho} \sum_a \rho_a e_a, $$

$$ T = \frac{1}{n} \sum_a n_a T_a, \quad n = \sum_a n_a. \quad (1.1) $$

Here $n_a$, $e_a$, and $m_a$ are, respectively, the average number of particles per unit volume, charge, and mass of $a$-type particles.

In MHD we study phenomena in which the electric field $E \equiv (u_0/c)B$ (E and B are the moduli of the electric field intensity vector and the magnetic induction vector; $u_0$ is the characteristic velocity of motion of the conducting medium). This inequality, along with the other assumptions, makes it possible to simplify considerably the system of equations describing the motion of the conducting medium within the framework of MHD [inequality (1.2) is satisfied], may be obtained in various ways. In the present paper we employ a method based on the kinetics equations written for each of the plasma components [7, 8].

We multiply the right and left sides of the kinetics equations by the quantities $m_a$, $m_a v_a$, and $0.5m_a v_a^2 + e_a$ ($v$ is the microscopic velocity of the particle, $e_a$ is the internal energy of $a$-type particles) and integrate over the entire velocity space. Taking in (1.2) the quantity $u_0 = \max \{u^a\}$, in the equations obtained after integration the magnetic field terms may be dropped by virtue of inequality (1.2).

The system of equations has the form

$$ \frac{\partial \rho_a}{\partial t} + \text{div} \rho_a u_a = R_a^{(i)}, \quad \sum_a R_a^{(s)} = 0, \quad (1.3) $$

$$ \frac{\partial}{\partial t} \rho_a u_a + \text{div} \left( \rho_a u_a u_a + \rho_0 \delta_{ij} - \rho_0 w_{ai} w_{aj} \right) + p_0 \delta_{ij} + \varepsilon \rho_0 \delta_{ij} - \rho_0 w_{ai} w_{aj} \right) - e_a n_a E_i = R_a^{(i)} + R_a^{(s)} u_i, \quad \sum_a R_a^{(i)} = 0, \quad (1.4) $$

$$ \frac{\partial}{\partial t} \rho_a E_a + \text{div} \Pi_a - e_a n_a u_a E = \frac{R_a^{(i)}}{\sum_a} + u R_a^{(s)} + 0.5 \varepsilon A R_a^{(s)}, \quad \sum_a R_a^{(s)} = 0. \quad (1.5) $$

$$ E_a = e_a + 0.5 u_a^2 - 0.5 w_a^2, $$

$$ \Pi_a = \rho_a u_a E_a + u p_a + u \cdot \pi_a + q_a + 0.5 \rho_a w_a $$

$$ p_a = \rho_a \left( p_a, \quad T_a \right), \quad \varepsilon_a = \varepsilon \left( T_a \right) \quad (1.6) $$

Here $\pi$ is the viscosity stress tensor, $w_a = u_a - u$ is the diffusion velocity, $q_a$ is the thermal flux, $e_a$ is the enthalpy per unit mass of the
Fluid Dynamics

The electric field is transformed, while the magnetic field is not. It is convenient to consider the equation for the energy of the system to be (1.2) is satisfied, we have

\[ \frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{u} = 0, \quad \frac{\partial}{\partial t} \mathbf{q} + \text{div} \mathbf{j} = 0, \]

\[ p = p(p, \rho, T), \quad \rho = \rho(p, T), \]

\[ \frac{\partial}{\partial t} \rho \mathbf{u} + \frac{\partial}{\partial x_i} (\rho u_i \mathbf{u}_j + p \delta_{ij} + \pi_{ij}) = \frac{q}{\epsilon}, \]

\[ \pi_{ij} = \sum_a \pi_{ij}^{(a)}, \]

\[ \frac{\partial}{\partial t} \rho \left( \varepsilon + \frac{\mathbf{u}^2}{2} \right) + \text{div} \left[ \rho \mathbf{u} \left( \mathbf{u} + \frac{\mathbf{u}^2}{2} \right) + \mathbf{u} \cdot \pi + q \right] = \mathbf{j} \cdot \mathbf{E}, \quad \mathbf{q} = \sum_a \mathbf{q}_a. \]

In solving problems it is convenient (particularly for use with the Maxwell equations) to use Eqs. (1.1) for the parameters, which characterize the plasma as a whole. These equations are obtained from (1.3)-(1.6) by summation over all \( \alpha \). Within the framework of EHD [12] we can show that the electric field is not transformed in virtue of (1.2), the electric field is transformed, while the magnetic field is not. It is easy to show that the EHD equations do not depend on the choice of the inertial coordinate system.

Let us evaluate the magnetic field terms in the Maxwell equations. Introducing the dimensionless quantities \( \mathbf{E}_0 = \mathbf{E}/\mathbf{E}_0 \) and \( \mathbf{r}_0 = \mathbf{r}/\mathbf{L} \) (\( \mathbf{E}_0 \) and \( \mathbf{L} \) are the characteristic values of the electric field and length) we obtain

\[ \text{rot} \mathbf{E} = \frac{\mathbf{e}_0}{c} \frac{\partial \mathbf{E}}{\partial t} + 4\pi \mathbf{q}/\mathbf{e}_0. \]

\[ \text{div} \mathbf{B} = 0, \quad \mathbf{j} = \sum_a \mathbf{q}_a \mathbf{u}_a. \]

and using the values of \( \mathbf{E} \) and \( \mathbf{j} \) obtained from the problem solution. We show that the variation of \( \mathbf{B} \), described by (1.8), satisfies (1.2). Taking the estimate \( \mathbf{j} \sim q \mathbf{u}_0 \) for the current density and using the connection between \( \mathbf{B} \) and \( \mathbf{H} \) in the moving medium, from the first equation of (1.8) we can evaluate the magnitude of the magnetic induction change \( \Delta \mathbf{B} \) over the characteristic length:

\[ u_0 \Delta \mathbf{B} / u_0 \mathbf{E}_0 \mathbf{E} \sim 4\pi \mathbf{q}_a \mathbf{E}_0 / \epsilon_0 \mathbf{E}_0 \mathbf{E} \sim (u_0^2 / \mathbf{E}_0) Q. \]

\[ (Q = 4\pi \mathbf{q}_a / \epsilon_0 \mathbf{E}_0). \]

Over a wide range of realistic conditions

\[ (u_0^2 / \mathbf{E}_0) Q \ll 1, \quad u_0 \Delta \mathbf{B} / u_0 \mathbf{E}_0 \ll \mathbf{E}. \]

Using the formulas for transformation of the electric and magnetic fields to convert from one inertial coordinate system to another, we can see that by virtue of (1.2) the electric field is not transformed in EHD; only the magnetic field is transformed. In MHD, conversely, only the electric field is transformed, while the magnetic field is not. It is easy to show that the EHD equations do not depend on the choice of the inertial coordinate system.