CONSTRUCTION OF LIQUID FLOW PAST A SOLID BODY WITH AN AIR LAYER

M. V. Tret'yakov


It is known that the friction drag for a body moving in a liquid may be reduced by varying the physical properties of the liquid at the body surface and, in particular, by creating an artificial air cavity on the body surface.

The question of the magnitude of the drag gain in the case of the presence of a wall fluid layer with different physical constants was considered in [1-3].

In [4, 5] results are presented of theoretical and experimental studies on the question of determining the parameters of artificial air cavities created on the lower surface of a horizontal wall in order to reduce the drag.

In the following an attempt is made to construct the three-dimensional flow of an ideal liquid past a body which is enveloped entirely in an air cavity.

1. FORMULATION OF THE PROBLEM

Let S be an unclosed—smooth in the Lyapunov sense—surface obtained by rotating the arc OC about the z-axis of the cylindrical coordinate system z, r, θ with origin at the point O (Fig. 1).

![Fig. 1](image)

We locate on this surface a layer of sources of intensity m(M) of an ideal fluid of density ρ and at the point B of the z-axis, at the distance h from the origin, we locate a sink of intensity Q.

If we now direct onto the surface (S) a translational stream of ideal fluid of the same density with the velocity $\text{v}_0$, directed along the z-axis in the opposite direction, the velocity potential of the flow which then occurs is written as

$$\Phi(P) = \text{v}_0 z - \frac{1}{4\pi} \int_0^1 \frac{m(M)}{r_1} dF + \frac{Q}{4\pi h}. \quad (1.1)$$

We know that the integral appearing here is the potential of a simple layer with density m(M).

We direct the normal to the surface (S) in the direction of the positive z-axis, and we seek the function m(M) of the intensity of the source layer so that the following boundary condition is satisfied on the surface (S):

$$\left(\frac{\partial \Phi}{\partial n}\right)_- = \sigma \left(\frac{\partial \Phi}{\partial n}\right)_+. \quad (1.2)$$

Here σ is a given constant quantity, and $\left(\frac{\partial \Phi}{\partial n}\right)_-$ and $\left(\frac{\partial \Phi}{\partial n}\right)_+$ are the limiting values of the normal components of the velocity of the considered flow at the points of the surface (S).

Taking account of the equations given in [6] for the limiting values of the normal derivative of the potential of a simple layer distributed over an unclosed surface, we find from (1.1) by differentiation the expressions for the limiting values of the normal components of the velocity of the radiated stream at the point $M_0$, and substituting them into the boundary condition (1.2) we obtain the integral equation

$$m(M_0) = -\frac{\lambda}{2\pi} \int \frac{m(M)}{a^2} dF + 2\text{v}_0 \cos \beta_0 + \frac{\lambda Q \cos \alpha}{2na^2} \left(\frac{1 - \sigma}{1 + \sigma}\right). \quad (1.3)$$

Here $\beta_0$ is the angle between the normal at the point $M_0$ and the z-axis. The solution of this integral equation yields the function $m(M_0)$ of the intensity of the source layer corresponding to the given value of the constant coefficient σ.

With account for (1.3), the limiting values of the normal components of the stream velocity at the points of the surface are expressed in terms of the layer intensity function $m(M_0)$ as

$$\left(\frac{\partial \Phi}{\partial n}\right)_\pm = \frac{1 \pm \lambda}{2\pi} m(M_0). \quad (1.4)$$

On the basis of the Bernoulli equation and (1.4), the pressure differential at the surface points is

$$\Delta p = \frac{1}{2}\rho \lambda^{-1} m^2(M_0). \quad (1.5)$$

Now let $-1 < \sigma < 0$. According to (1.4) the normal component $\left(\frac{\partial \Phi}{\partial n}\right)_-$ will be directed opposite $\text{v}_0$, and, consequently, on the z-axis ahead of the surface (S) there will be a stagnation point D at which the stream velocity is zero. The coordinate $z_0$ of this point is determined from the condition $\left(\frac{\partial \Phi}{\partial z}\right)_0 = 0$ for $z = z_0$, and through this point passes the stream surface L which separates the incident outer stream from the inner stream created by the source layer. The equation of this stream surface L will be

$$\int_{z=0}^{z_0} \left(r \frac{\partial \Phi}{\partial r}\right) dz - \left(r \frac{\partial \Phi}{\partial z}\right) dr = 0. \quad (1.6)$$

We select the sink intensity Q at the point B so that all the liquid fluid emanating from the source layer in the positive direction of the normal to the surface (S) is removed by this sink, i.e., so that

$$\int_{z=0}^{z_0} \left(\frac{\partial \Phi}{\partial n}\right)_+ = Q. \quad (1.6)$$

Now let us replace the source layer of fluid with density ρ by a source layer of some other fluid with
density $\rho_1$, but we select the intensity of the radiation of this layer and the strength of the sink at the point B so that the interface L of the outer and inner flows is not changed. Then the outer flow potential is expressed by the same equation (1.1). Both the outer and inner flows are potential and irrotational. Therefore, considering that the pressures of the two flows are equal at the point D, we can show [6] that the velocity potential of the inner flow is

$$\Phi_1(p) = k\Phi(p) \quad (k = \sqrt{\frac{\rho_0}{\rho_1}} = \text{const}).$$  \hfill (1.7)

The basic quantities of the inner flow are

$$v_1 = kv_0, \quad m_1(M) = km(M), \quad Q_1 = kQ.$$  \hfill (1.8)

Condition (1.6) for the inner flow remains in force, and, consequently, on the z-axis there will be a stagnation point A at which the flow velocity will be zero. The coordinate $z_1$ of this point is determined from the condition $(\partial \Phi_1/\partial z) = 0$ for $z = z_1$, and through this point passes the stream surface $L_1$, whose equation is

$$\int_{(S)} \left( r \cdot \frac{\partial \Phi_1}{\partial r} \right) dz - \left( r \cdot \frac{\partial \Phi_1}{\partial z} \right) dr = 0.$$  \hfill (1.9)

This surface together with the surface (S) forms a closed surface of revolution. Replacing this stream surface $L_1$ by a solid wall, we obtain the flow pattern about a solid body of revolution whose leading end has the form of the surface (S), and from each point of this surface, fluid with density $\rho_1$ emanates with a normal flow velocity component $(\partial \Phi_1/\partial n)_c$, and the flow-rate of this emanating stream is

$$\int_{(S)} \left( r \cdot \frac{\partial \Phi_1}{\partial n} \right)_c dF = N_1.$$  \hfill (1.10)

This stream of fluid of density $\rho_1$ then creates an interlayer between the body surface and the zero stream surface $L$ of the outer fluid flow of density $\rho$. In particular, the fluid of density $\rho_1$ may be air.

The case $\sigma = -1$ cannot occur physically, since in the presence of an approaching flow the strength of the layer sources in this case must be infinitely great. In addition, the interlayer of fluid of density $\rho_1$ on the body must be sufficiently thin. Therefore, the coefficient $\sigma$ must satisfy the condition $-1 < \sigma < 0$, which ensures uniqueness of the solution of the basic equation (1.3).

2. SOLUTION OF THE PROBLEM FOR A HEMISPHERE

Let the surface (S) be a hemisphere of radius R (Fig. 2). We take the coordinate origin at the center O of the hemisphere, and we denote the distance $OB$ by $h$. Then we obtain directly from the sketch (Fig. 2)

$$\cos \theta = \frac{1 - b \cos \varphi_0}{R^2(1 + b^2 - 2b \cos \varphi_0)^{n/2}} \left( b = \frac{h}{R} \right).$$

Now let us convert to spherical coordinates:

$$x = R \sin \varphi \cos \theta, \quad y = R \sin \varphi \sin \theta, \quad z = R \cos \varphi.$$

Considering the symmetry of the flow relative to the z-axis, the equality $\cos \varphi = a/R$, and the expressions for $dF$ and $a$ in terms of $R$, we find that the basic equation (1.3) for this case has the form

$$m(\varphi_0) = -\frac{\lambda}{4\pi} \int_{(S)} m(\varphi) \frac{dF}{a} - 2\lambda v_0 \cos \varphi_0 + \frac{\lambda Q}{2\pi} \frac{1 - b \cos \varphi_0}{R^2(1 + b^2 - 2b \cos \varphi_0)^{n/2}}.$$  \hfill (2.1)

Here the integration is performed over the hemisphere (S) of unit radius. We shall consider that Fig. 2 is a hemisphere of unit radius.

If we denote the distance from the point P, lying on the radius $OM_0$, to the center O by $\eta$, the distance $PM$ by $\alpha$, and the angle between OP and OM by $\gamma$, then, as is known [7], we have the expansion

$$A_\alpha = \frac{1}{\eta^4 - 2\eta \cos \gamma + \eta^2} = \sum_{n=0}^\infty \eta^n X_n(\cos \gamma).$$  \hfill (2.2)

Here $X_n(\cos \gamma)$ is a Legendre polynomial of degree $n$. Since $\eta < 1$, series (2.2) converges uniformly.

Multiplying (2.2) by $m(\varphi_0)$ and integrating over the hemisphere of unit radius, we obtain

$$\int_{(S)} \int_{(S)} m(\varphi) \frac{m(\psi)}{a} dF = \sum_{n=0}^\infty \eta^n \int_{(S)} m(\varphi) X_n(\cos \varphi) dF.$$  \hfill (2.3)

But $\cos \gamma = \cos \varphi_0 \cos \varphi + \sin \varphi_0 \sin \varphi \cos (\theta - \theta_0)$, and on the basis of the theorem on addition of Legendre polynomials we shall have

$$X_n(\cos \gamma) = X_n(\cos \varphi) X_n(\cos \varphi_0) + 2 \sum_{m=1}^n \frac{(n - m)!}{(n + m)!} X_n^m(\cos \varphi) X_n^{m^*}(\cos \varphi_0) X_n m(\cos \varphi) m(\cos \varphi_0) m(\cos \theta) (\theta - \theta_0).$$  \hfill (2.4)

Here $X_n^m(\cos \varphi)$ is the Legendre associated function. Substituting (2.4) into (2.3), making the corresponding calculations, and passing to the limit as the point P approaches the point $M_0$ along the radius (here $\eta \to 1$), we obtain

$$\int_{(S)} \int_{(S)} m(\varphi) \frac{m(\psi)}{a} dF = -2\pi \sum_{n=0}^\infty X_n(\cos \varphi_0) \int_{(S)} m(\varphi) X_n(\cos \varphi) d(\cos \varphi).$$