CALCULATING THE EFFECT OF GUSTS ON AN ARBITRARY THIN WING

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A numerical method of calculating the unsteady flow about a thin wing moving in an ideal incompressible medium is developed on the basis of the lifting surface scheme. The variation of the boundary conditions on the wing surface with time and coordinates may be arbitrary. Therefore, the method makes it possible to examine the aperiodic motion of a wing as a rigid body, consider any wing deformations, analyze the wing entry into a gust, study the effect of a weak shock wave on the wing, etc. In addition, practically no limitation is imposed on the shape of the thin lifting surface: the method is applicable to monoplane wings of any planform, to annular wings, to systems of similar wings, etc.

Studies in which the effect of a gust on a wing is analyzed have been reviewed in [1, 2]. Without dwelling on this review, we note that at subsonic speeds an effective solution of the problem has been obtained only for a profile.

§1. Basic principles of the method. Assume that any arbitrary thin lifting surface executes unsteady motion as a rigid body, or is deformed, or enters suddenly or gradually into a gust. Knowing the gust characteristics, the laws of motion, and the body deformations, it is not difficult to determine the corresponding normal velocity components on the wing surface, which may thus be considered known. The normal component $W_\eta$ of the disturbed velocity associated with motion of the lifting surface must compensate them at any instant of time (smooth flow condition). We note that within the framework of the linear theory considered below the gust action, the wing motion as a rigid body, and the wing surface deformations may be studied independently.

For definiteness, we will consider a monoplane flat-plate wing of arbitrary, but symmetrical planform. We introduce the Oxyz body coordinate system with origin at the midpoint $b$ of the root chord (Fig. 1). The basic time-independent wing velocity $U_0$ is assumed to be directed along the root chord (Ox axis).

In the general case, the boundary conditions on the wing may be represented in the form

$$\frac{W_\eta}{U_0} = cf\left(\frac{x_0}{b}, \frac{z_0}{b}, \tau\right),$$  

$$\tau = \frac{U_0 t}{b}.$$  

(1.1)

Here $x_0$, $z_0$ are coordinates of a point on the wing surface, $\tau$ is dimensionless time, and $c$ is a normalizing dimensionless constant. For example, for bounded right sides of (1.1) it is advisable to select $c$ so that $|f| \leq 1$. In the linear problem we can consider that with $\tau < 0$, $f = 0$. We note also that if the rigid flat-plate wing moves at zero angle of attack and gradually enters a sharp-edged gust, then on the part of the wing that has not entered the gust the function $f = 0$.

The disturbed motion of the fluid away from the wing and the vortex trail behind it may be considered potential. The disturbed velocity potential will satisfy the Laplace equation for both the steady and unsteady motions. On the vortex sheet behind the wing, the pressure must vary continuously, and, at the
trailing edges of the wing, the Chaplygin-Zhukovskii condition must be satisfied.

The pressure difference $\Delta p$ on the lower and upper wing surfaces for arbitrary unsteady motion may be expressed in terms of the intensity of the bound vortex layer $\gamma_+$ on the basis of the Zhukovskii theorem "in the small" [3]:

$$\Delta p = -\rho \gamma_+ W_{0n}.$$  

(1.2)

Here $W_{0n}$ is the relative velocity component of the medium normal to the axis of the vortex $\gamma_+$ at a point on the vortex layer, and $\rho$ is the medium density.

As is known, the velocity potential corresponding to a vortex surface or to discrete vortices satisfies the Laplace equation. According to (1.2), if the normal component of the relative velocity of the medium is equal to zero, there will be no pressure difference on the free surface. Consequently, the axes of the free vortices must be directed along the local velocity of the medium, or these vortices must move with a velocity equal to the medium velocity. Within the framework of the linear theory we can take this velocity as equal to $U_0$; then in place of (1.2) we have

$$\Delta p = \rho \gamma_+ U_0.$$  

(1.3)

$\gamma_+ z$ is used to denote the intensity of the bound vortices whose axes are perpendicular to the velocity $U_0$. The positive directions of $\Delta p$ and $\gamma_+ z$ are shown in Fig. 1. Thus, if the wing is replaced by a vortex layer, the series of problem conditions is satisfied automatically. It is necessary only to select the intensity of this layer so as to satisfy the smooth flow and Chaplygin-Zhukovskii conditions.

The basic idea of the numerical method of solving this problem, which will be discussed below, involves transformation from continuous to discrete distributions and processes.

First, the continuously distributed vortex layer, which replaces the wing, is simulated approximately by a system of discrete vortices. However, in contrast with the way this is done in [3, 4], this substitution is made not for the bound vortex layer, but for the combined layer consisting of the free and bound vortices on the wing and the free vortices behind the wing (Fig. 2).

Second, the continuous process of variation of the boundary condition and the circulation in time is replaced by a step process. At definite instants of time there are step changes of the boundary conditions and the circulations while in the interval between these changes the circulations do not vary. Thus, the free vortices separate from the bound at discrete instants of time (Fig. 3).

§2. Vortex model of wing. For monoplane wings the basic elementary vortex systems will be skewed horseshoe vortices (Fig. 4). The circulation of the transverse vortex and of the longitudinal (parallel to the velocity $U_0$), semi-infinite vortices are constant and equal to $\Gamma_\ast$. According to [3], the velocity induced by this vortex system in the $x\ast$, $z\ast$ plane may be represented in the form

$$W_\nu = \frac{U_\nu}{z \nu} w_\nu(\xi_0, \tau_\nu, \chi),$$

$$\Gamma_\ast = U_\nu \nu \Gamma_\nu, \quad \xi_\nu = \frac{2\gamma_\nu \mu}{\nu}, \quad \tau_\nu = \frac{2\nu}{\nu}.$$  

(2.1)

$w_\nu(\xi_0, \tau_\nu, \chi)$ represents the dimensionless function for which an expression was given in [3].

The transition from continuous variation of the circulation in time to discrete variation makes it possible to consider the indicated stationary vortex as the basic elementary system. Suppose that on an arbitrary element of the wing circulation of the bound vortex $\Gamma_\ast$ changes at some instant of time $t$ by an amount $\Delta \Gamma_\ast$. This will be accompanied by the shedding of a transverse bound vortex of circulation $\Delta \Gamma_\ast$. At the instant of time $t + \Delta t$, in place of the skewed stationary vortex (Fig. 4), we get the vortex system shown in Fig. 5.

We note that the closed vortex filament $ABCD$ of constant circulation $\Delta \Gamma_\ast$ may be considered as the sum of the two transverse vortices $AB$ and $CD$ of equal and opposite circulation with their corresponding semi-infinite longitudinal vortices.

In replacing the continuous vortical sheet by discrete vortices directly on the wing, it is advisable to proceed as in the stationary case [3] and the problem on the harmonic oscillations of a wing [4].

In Fig. 2 the wavy lines show the vortices, and the crosses denote the index points at which the boundary conditions are satisfied. We will designate the numbers of the transverse vortices by $i$, the computational points by $j$, the vortex filaments by $\mu$, the reference lines by $\nu$, and the sections parallel to the $Ox$ axis by $k$. With this arrangement it is essential that, first, the reference points lie at the midpoint between the vortices nearest to them. Second, in each strip $k$ the last reference point lies closer to the tail than the last vortex.

Generally speaking, the required number of vortices on the wing