QUANTIZATION OF A PARTICLE-LIKE CLASSICAL SOLUTION IN BOGOLIUBOV VARIABLES WITHOUT DIRAC’S BRACKETS

O. A. Khrustalev and K. A. Sveshnikov

The Bogoliubov collective coordinate method is applied to soliton quantization in such a way that the soliton and the meson field are considered as independent canonical degrees of freedom of the system from the beginning. The quantization of the enlarged system proceeds by means of canonical rules without applying to Dirac’s formalism of constrained systems. The true number of independent degrees of freedom is fixed by means of a set of additional conserved charges and corresponding symmetry relations. The role of the soliton recoil in the dynamics of the coupled system is discussed in detail. The full form of the Born amplitudes in the meson-soliton scattering is calculated directly in the Hamiltonian picture. The relations to other methods of soliton quantization are also discussed.

In memory of N. N. Bogoliubov

1. INTRODUCTION

Quantization in the neighborhood of a particle-like classical solution is an important tool in studying nonlinear phenomena associated with different kinds of extended objects in particle and condensed matter physics. Several different approaches have been developed for this problem since the outset (see [1] for a review). Among them, the method of collective variables, introduced by N. N. Bogoliubov in the polaron problem in 1950, turns out to be the most effective and popular one. After the pioneering work of Bogoliubov [2], the efficiency of this method has been demonstrated in numerous series of papers concerning systems with strong coupling [3–5], canonical quantization of solitons [6–8] and other particle-like classical solutions such as skyrmions [9, 10], etc.

An attractive feature of this method is that it allows for a self-consistent quantum-mechanical treatment of those degrees of freedom of the system that are inherent to the classical extended object itself and are essentially different from the usual field quanta. These are position coordinates, (iso)rotational, scaling, and other free dynamical parameters, contained in the classical solution. In the conventional framework of the collective coordinate method (CCM) these modes are promoted to be the group degrees of freedom of the whole system in order to restore broken symmetries and to obtain states with physical quantum numbers. A typical such example is the recent development of the baryon skyrmion model [10].

A delicate feature inherent to the conventional CCM is that the secondly quantized (“meson”) fields are considered in this approach in the rest frame of the extended object (“soliton”). Usually one assumes that the recoil effects are negligible, and then to the leading order one obtains the meson dynamics in the static soliton background. When the initial fields are expanded in the vicinity of the classical solution, this approach provides a systematic perturbation theory within the Dirac’s framework of constrained systems [11] without zero modes and corresponding divergencies. However, physically the picture where the classical objects coexist with quanta and interact with them seems to be more realistic. Then one has to consider the soliton and mesonic variables as independent canonical degrees of freedom of the system from the beginning.

The purpose of this paper is to consider a modification of the Bogoliubov method of such kind. In contrast to the conventional CCM, the introduction of the additional soliton variables in this approach
does not lead to constraints. The quantization of the enlarged system proceeds then by means of canonical quantization rules without applying to Dirac’s formalism. The constrained nature of the dynamics of the enlarged system shows up in the fact that the system admits a corresponding number of additional integrals of motion. Thus, the true number of the independent degrees of freedom is conserved. The conventional CCM formulas are reproduced in this approach as a particular case of the choice of reference frame.

The paper is organized as follows. In Sect. 2 the conventional method of soliton quantization via collective coordinate is reconsidered, the modified framework is presented, and the relations to other methods of soliton quantization are discussed. The dynamics of the coupled meson–soliton system in terms of the enlarged canonical space of states is investigated in Sect. 3. The role of the soliton recoil in the dynamics of the coupled system is also discussed in detail. In Sect. 4 the same approach is applied to the restoration of the full form of the Born amplitudes in the meson–soliton scattering directly in the Hamiltonian picture. In Sect. 5 we show how the same calculation should be implemented in the conventional context. Section 6 is devoted to concluding remarks.

2. SOLITON QUANTIZATION IN INDEPENDENT VARIABLES

In order to avoid unnecessary complications, we consider the generic 1 + 1-dimensional model of a nonlinear scalar field \( \varphi(x, t) \), described by the Lagrangian density

\[
\mathcal{L}(\varphi) = \frac{1}{2} \left( \partial_\mu \varphi \right)^2 - U(\varphi) ,
\]

which possesses a static soliton solution of the form \( \varphi_c(x) = u(x - X) \), where \( X \) is a parameter.

In the CCM, in the vicinity of \( \varphi_c(x) \) the Heisenberg field \( \varphi(x, t) \) is decomposed into a new set of Heisenberg operators, namely, the position operator \( X(t) \) and the meson field \( \cosh i(x, t) \), in the following way [6–8]:

\[
\varphi(x, t) = \varphi(x - x(t)) + x(\varphi - x(t), t) .
\]

As a result the shift \( X(t) \rightarrow X(t) + a \) induces a spatial translation of \( \varphi(x, t) \), and the momentum \( P(t) \) conjugated to \( X(t) \) coincides with the total momentum of the system

\[
P(t) = - \int dx \varphi'(x, t) \pi(x, t) .
\]

Therefore, the soliton degree of freedom is promoted to provide the symmetry property of the whole field. As a consequence, the relation (3) considered in new variables leads to a first class constraint, which is resolved within the Dirac’s framework of constrained systems [6–8, 12]. Within this approach the decomposition (2) is accomplished by a corresponding second class constraint (subsidiary condition), conjugated to the constraint (3) in the Dirac sense. In the literature [2–8, 12] one has used almost exclusively the linear condition of the form

\[
\int dx N(x) \chi(x, t) = 0 ,
\]

where it is useful to normalize \( N(x) \) so that

\[
\int dx N(x) u'(x) = 1 .
\]

In this case the canonical momentum \( \pi(x, t) \) is decomposed as [6–8]

\[
\pi(x, t) = \eta(x - X(t), t) = \frac{1}{2} \left\{ \frac{N(x - X(t))}{A(t)}, P(t) + \int dz \chi'(z, t) \eta(z, t) \right\} ,
\]

where

\[
A(t) = 1 - \int dy N'(y) \chi(y, t) ,
\]

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