FLOW PAST A SPHERE OF TWO CO-AXIAL SUPersonic STREAMS

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We investigate the flow past a sphere of a parallel supersonic stream which is nonuniform in magnitude; such a flow can be considered as two co-axial streams of an ideal gas. The problem is solved numerically by the method of establishment [1]. Supersonic flow of nonuniform magnitude and direction past blunt bodies and a plane wall was investigated in [2-5].

In what follows we shall indicate the parameters of the internal and external layers by the subscripts 1 and 2 respectively. The case when the velocity \( V_{\infty 1} \) of the inner stream is greater than the velocity \( V_{\infty 2} \) of the outer stream, as well as the case when \( V_{\infty 1} < V_{\infty 2} \) is computed. In the calculations the linear dimensions are related to the radius of the sphere, the velocity to the limiting velocity \( V_{\text{max}} \), the pressure to \( \rho_{\infty 1} V_{\text{max}}^2 \), and the density to \( \rho_{\infty 1} \).

There is a transitional zone between the streams.

The velocity profile \( V_{\infty} \) of the transitional zone is defined by the equation [6]

\[
\frac{V_{\infty 1} - V_{\infty}}{V_{\infty 2} - V_{\infty 1}} = (1 - \eta^2) \quad \text{for} \quad V_{\infty 1} > V_{\infty 2} \quad (1)
\]

\[
\frac{V_{\infty 2} - V_{\infty}}{V_{\infty 2} - V_{\infty 1}} = (1 - \eta^2) \quad \text{for} \quad V_{\infty 2} > V_{\infty 1} \quad (2)
\]

Here the dimensionless ordinate \( \eta \) (it refers to the width of the transitional zone) is measured from the upper boundary of the transitional zone for the first case and from the lower boundary for the second case. The velocity profile at the meridional section is shown in Fig. 1a for \( M_{\infty 1} = 3.0, M_{\infty 2} = 2.0 \).

The static pressure \( p_{\infty} \) is constant through the whole incident flow and is defined by the equation

\[
\frac{p_{\infty}}{p_{\infty} V_{\text{max}}^2} = \frac{1}{2} \left[ 1 - \left( \frac{V_{\infty 1}}{V_{\text{max}}} \right)^2 \right] \quad (3)
\]

For the density \( \rho_{\infty} \) and the velocity \( V_{\infty} \) we have

\[
\frac{\rho_{\infty}}{\rho_{\infty 1}} = \frac{1 - \left( V_{\infty 1}/V_{\text{max}} \right)^2}{1 - \left( V_{\infty 1}/V_{\text{max}} \right)^2}, \quad \frac{V_{\infty}}{V_{\text{max}}} = \left[ \frac{M_{\infty 1}^{\gamma-1}}{M_{\text{max}}^{\gamma-1} + 1} \right]^{\frac{1}{\gamma-1}} \quad (4)
\]

The radius of the outer stream is assumed to be so large that the outer boundary of the stream has no effect on the region of the flow to be computed. The radius \( R_1 \) of the inner stream is chosen so that it is less than the distance from the axis of the sonic point on the shock wave for uniform flow past a sphere with \( M_{\infty} = M_{\infty 1} \).

Thus, the incident flow is defined by five parameters: the Mach numbers of the inner and outer streams \( M_{\infty 1} \) and \( M_{\infty 2} \), the adiabatic index \( \gamma \), the radius of the inner stream \( R_1 \) and the width of the transitional zone \( h \). All the calculations, the results of which are given below, were carried out for \( \gamma = 1.4, R_1 = 0.4, h = 0.2 \).

The geometrical pattern of the flow of two co-axial streams past a sphere is shown in Fig. 1b (for $M_{\infty 1} = 3.0$, $M_{\infty 2} = 2.0$). Here AM is the shock wave, BGD the sonic line, CGF the limiting characteristic. The flow is computed in the region AMNE, the boundary line MN being chosen so that the velocity component normal to that line is greater than the speed of sound $V_n > a$; in this case no perturbations are transmitted from the region which is below the flow with respect to MN to the region AMNE and no boundary conditions can be formulated on the line MN.

A polar coordinate system $r, \theta$ is used on the meridional plane. Across the shock layer we introduce a normalized coordinate $\xi$, defined by the equation

$$\xi = \frac{r - r_w(\theta)}{r_s(\theta) - r_w(\theta)}$$

(5)

Here $r_w = r_w(\theta)$ and $r_s = r_s(\theta)$ are the equations of the contour of the body and of the shock wave. The difference mesh has $11 \times 21$ points. The step size in the $\xi$ direction is $h_\xi = 0.1$, and the step size in the $\theta$ direction is $h_\theta = 0.05 \theta'$, where $\theta'$ is the coordinate of the boundary ray.

Consider first the case when $V_{\infty 1} > V_{\infty 2}$.

The geometrical patterns of the flow for a range of moderate and small supersonic velocities are shown in Fig. 2a and b. The curves in these illustrations correspond to the following Mach numbers: 2($M_{\infty 1} = 3.0$, $M_{\infty 2} = 2.5$), 4($M_{\infty 1} = 3.0$, $M_{\infty 2} = 2.0$), 6($M_{\infty 1} = 1.5$, $M_{\infty 2} = 1.4$), 7($M_{\infty 1} = 1.5$, $M_{\infty 2} = 1.3$). For comparison the shock waves and sonic lines are indicated by dotted lines for a uniform flow past a sphere: the curves 1, 3, 5 correspond to $M_{\infty} = 3.0$, 2.0, 1.5.

We note that for large supersonic velocities the geometrical pattern of the flow has the same nature as for moderate supersonic velocities. As we see from the illustrations, the shock wave and sonic line have more complex forms for the flows under consideration than for uniform flow past a sphere. In the transition region the shock waves have two points of inflection, the inflection of the shock wave becoming more distinct as $M_{\infty 2}$ falls at constant $M_{\infty 1}$. As for the sonic line, it should be noted that as $M_{\infty 2}$ falls the sonic point on the body is displaced towards the axis while the sonic point on the shock wave moves upwards. At moderate and large supersonic velocities the sonic line has two points of inflection.

An important variable determining the position of the shock wave is its distance from the axis. As $M_{\infty 2}$ decreases at constant $M_{\infty 1}$, this variable increases, most strongly for small supersonic velocities,