Abelian chromodynamics is investigated using the singular infrared asymptotic behavior \((k^2)^{-2}\) of the gluon propagator. The method of functional integration is used to introduce bilocal variables that describe the meson fields. It is shown that in the model there is dynamical breaking of the chiral symmetry, the pions playing the part of the Goldstone bosons. Expressions are obtained for the dependence of the quark constituent mass and the magnitude of the quark condensate on the momentum cutoff that determines the meson form factor. Calculations are made of the meson mass spectrum and the total effective action that describes the interaction of the scalar and pseudoscalar mesons. For cutoff momentum \(\Lambda = 1\) GeV and quark current mass \(m_0 = 5\) MeV the calculated pion mass is \(m_\pi = 140\) MeV, the dynamical quark mass is \(m = 241\) MeV, and the quark condensate is \(\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(248\) MeV\(^3\).

1. Introduction

Quantum chromodynamics (QCD) describes the interaction of quarks and gluons at high energies in the region of asymptotic freedom. Here, perturbation theory is valid and the Feynman diagram technique "works." At low energies there are nonperturbative effects and perturbation calculations are impossible because of the large value of the quark-gluon coupling constant.

To prove the universality of QCD, we must know how to make calculations at low temperatures, explain the nonperturbative effects — the breaking of chiral symmetry, quark confinement, etc. — and obtain effective chiral Lagrangians that describe hadronic physics adequately. This, in its turn, requires a reformulation of QCD in terms of hadronic degrees of freedom.

An important step in this direction was made in [1], in which a computational scheme for large \(N_c\) (\(N_c\) is the number of quark colors) was developed under the assumption of breaking of chiral symmetry. There are also other approaches to this problem [2-9].

Breaking of chiral symmetry in field theory was first demonstrated at the model level in [10-12]. A topical problem is that of explaining chiral symmetry breaking in the framework of QCD. This breaking is manifested in the appearance of a quark condensate. Phenomenology gives for the vacuum expectation values \(\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(249-250\) MeV\(^3\). If the quark current masses are zero, pions play the part of Goldstone bosons.

To simplify the treatment, we shall proceed from Abelian chromodynamics [13]. The non-Abelian nature of QCD and the gluon self-interaction are here taken into account by using a singular infrared asymptotic behavior of the gluon propagator [14,15]. The considered gluon propagator is a nonperturbative solution of the Schwinger-Dyson equations. When such a simplification is made, there are none of the difficulties associated with the gluon self-interaction and the presence of "ghosts." However, the Ward-Slavnov-Taylor identities have the same form as in QCD [13].

Our approach differs from [3-6], in which the ordinary gluon propagator, and not the "infrared" one, was used.

We shall proceed from a model motivated by QCD with action

\[
S = \int dx\, dy\, (-\bar{\psi}(x)(\gamma_\mu(\partial_\mu-i g A_\mu)\psi(x)\delta(x-y) - \gamma^5 A_\mu(x)D^{\mu\nu}(x,y))^{-1}A_\nu(y) \right),
\]
where $m_0i$ are the quark current masses, $A^a_\mu(x)$ are the gluon fields, and $\lambda^a$ are Gell-Mann matrices acting on the space of colors with $N_c = 3$. In (1) there is summation over the quark flavors $i = 1, 2, \ldots, N_f$. Here, we use the gluon propagator in the infrared region [14-15]:

$$D^\mu\nu(x, y) = \delta_{\mu\nu} D^\mu(x, y) = \delta_{\mu\nu} x^2 \left[ \frac{(x_\mu - y_\mu)(x_\nu - y_\nu)}{(x_\mu - y_\mu)^2} - \beta \delta_{\mu\nu} \right].$$

(2)

where $x^2$ is a parameter with the dimensions of a square of a mass.

The use of the propagator (2) automatically takes into account the nonperturbative effect of the gluon self-interaction.

In the momentum space, the gluon propagator (2) gives the singular infrared asymptotic behavior $(k^2)^{-2}$.

Note that in the considered approximation for the quarks in (1) we use the free propagator with the bare quark masses.

It will be shown below that as a result of the rearrangement of the physical vacuum the quarks acquire dynamical masses. Even if the quark current masses are zero ($m_0i = 0$), a state with massive quarks is energetically more advantageous. As a result, there is breaking of the chiral symmetry.

We shall assume that at the low energies we consider the coupling constant $g_0$ is constant (see [16]), so that the infrared pole in the coupling constant is eliminated.

The aim of the paper is to investigate the possibility of chiral symmetry breaking (for $m_0i = 0$) in the model based on the use of the action (1), motivated by QCD, and also to find the effective action in terms of the meson degrees of freedom.

In Sec. 2, we shall introduce bilocal variables and consider perturbation theory. We shall show that the chiral symmetry is broken and a quark condensate is formed. In Sec. 3, we find the masses of the lightest scalar and pseudoscalar mesons and the dynamical quark masses, and we also determine the value of the condensate and the quark masses as functions of the cutoff momentum.

In Sec. 4, we calculate the effective action for the meson fields in the case of two flavors.

2. Bilocal Variables and Loop Expansion

We use the representation of the generating functional for the Green's function in the form of a functional integral. Bearing in mind that the action (1) is quadratic in the gluon fields, we can perform a Gaussian integration, finding

$$Z[\eta, \eta] = \int D\bar{\psi} D\psi \exp[i(S + \bar{\psi}\eta + \eta\psi)],$$

(3)

$$S = \int d^4x d^4y \left[ -\bar{\psi}(x) (i\gamma_\mu \partial_\mu + m_{0i}) \psi(x) \delta(x - y) - \frac{g_0^2}{2} \bar{\psi}(x) \gamma^a \psi(x) D^\mu(x, y) \bar{\psi}(y) \gamma_a \psi(y) \right].$$

We note that an integral in the form (3) was used for a different form of gluon propagator in [3-6].

To introduce bilocal fields [17-19], we used generalized Fierz transformations. Bearing in mind that the gluon propagator (2) is symmetric in its indices, we find the symmetric part for the direct product of Dirac matrices (see [20]):

$$\gamma_\mu \otimes \gamma_\nu = \frac{1}{2} \left[ \gamma_\mu \otimes \gamma_\nu + \gamma_\nu \otimes \gamma_\mu \right] + \frac{1}{2} \delta_{\mu\nu} \gamma_5 \otimes \gamma_5 - \frac{1}{4} \delta_{\mu\nu} \gamma_5 \otimes \gamma_5 + \frac{1}{2} \delta_{\mu\nu} \gamma_5 \otimes \gamma_5 - \frac{1}{4} \delta_{\mu\nu} \gamma_5 \otimes \gamma_5 - 2 \delta_{\mu\nu} \gamma_5 \otimes \gamma_5 + \frac{1}{2} \delta_{\mu\nu} \gamma_5 \otimes \gamma_5,$$

(4)

where, as usual, summation is understood over repeated indices, $I_{(4)}$ is the unit matrix, and the round brackets indicate the symmetric part. We shall also need the direct products of the matrices in the color and flavor spaces [3-5]:

$$\lambda^a \otimes \lambda^b = \frac{1}{2} \delta_{ab} I_{(4)} \otimes I_{(4)} - \frac{1}{2} \lambda^a \otimes \lambda^b, \quad I_{(4)} \otimes I_{(4)} = \frac{1}{N_f} I_{(4)} \otimes I_{(4)} + 2 \lambda^a \otimes T^a,$$

(5)

where $T^a$ are the generators of the group SU($N_f$) acting on the flavor space.