The state (36) describes particles with spin $S$ and mass $M^2 = 2\pi S + M_0^2$, i.e., particles on a linear Regge trajectory.

5. Discussion

In this paper, we have proposed a quantum theory of a relativistic string in four-dimensional space. Note that the gauge conditions (12) and (21), which played a key part in our theory, are not specific for four-dimensional space alone. Therefore, our theory could be readily generalized to a space of an arbitrary number of dimensions.

In contrast to the standard approach, in which the light cone gauge is used, no problems arise here with operator ordering or relativistic invariance of the theory. Such a radical change of the theory on the transition from one gauge to another should not cause surprise. For in the classical theory the phase spaces corresponding to different gauges are related by a canonical transformation, and therefore the formulations of the theory in different gauges are physically equivalent. However, the group of nonlinear canonical transformations does not extend to the quantum theory, since on the transition to the latter the problem of operator ordering arises. Therefore, different quantum theories can correspond to different gauges of a given classical theory.

LITERATURE CITED


QUANTUM MECHANICS IN RIEMANNIAN SPACETIME.

I. GENERALLY COVARIANT SCHRÖDINGER EQUATION WITH RELATIVISTIC CORRECTIONS

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A study is made of the $c^{-2}$ asymptotics ($c$ is the speed of light) of the theory of a complex scalar field in a general Riemannian spacetime; the field interacts with an external electromagnetic field. In a freely falling (Gaussian normal) frame of reference we obtain a generally covariant analog of the Schrödinger equation for a scalar particle in external gravitational and electromagnetic fields with relativistic corrections of arbitrary order. It is shown that allowance for the geometrical variation in time of the phase-space element leads to a Hamiltonian that is (asymptotically) Hermitian with respect to the standard scalar product, and this provides a basis for the Born interpretation of the corresponding wave functions.

1. Introduction

The generalization of the (nonrelativistic) quantum mechanics of a particle to a general Riemannian spacetime (we denote it by $V_{1,3}$) with metric tensor $g_{\alpha\beta}$, $\alpha, \beta, \ldots = 0, 1, 2, 3$, is desirable from several points of view.

First, such a theory has applied importance for the study of quantum effects associated with an external gravitational field and noninertial motion, for example, in astrophysical processes in gravitational-wave experiments. But if the gravitational fields and noninertiality are so strong that particle creation processes become important, then one must go over to quantum field theory, and in this case it is important to define what is a
particle. It is evident that this is done most satisfactorily on the basis of Born's probability interpretation of the single-particle wave function.

The problem is also very important for deeper understanding of the structure of quantum mechanics on the basis of an extension of its geometrical foundation. Standard nonrelativistic quantum mechanics is usually regarded as an independent theory that, basically, admits an adequate observational interpretation. But in fact, this theory is only the limiting case \( c^{-2} = 0, G = 0 \) (\( G \) is the gravitational constant) of the as yet unknown complete theory of fundamental interactions in which the three universal constants \( \hbar, c^{-2}, \) and \( G \) occur without any simplifying assumptions. From the point of view of the study of approaches to this complete theory, it is natural to attempt to extend a theory that is so fully developed as quantum mechanics to the greatest possible neighborhood of the limiting point \( c^{-2} = 0, G = 0 \), which corresponds to a Galilean structure of spacetime and global inertial frames of reference.

The existing results in this direction relate mainly to quantum-mechanical systems in gravitational fields of a special form and use particular coordinate systems and superficial analogies with the standard theory (without gravitation). An extensive bibliography of these studies can be found in the monograph [1]. Among the general approaches, we mention the paper [2] of Kuchar, who formulates quantum mechanics in Newton-Cartan spacetime, which makes it possible to treat a nonrelativistic quantum system in a Newtonian gravitational field and in noninertial frames of reference. To a certain extent, this removes the condition \( G = 0 \). The introduction into quantum mechanics of a hyperbolic Riemannian spacetime structure leads to the loss of the most important concept of nonrelativistic physics — the absolute time. The concept of time now depends on the choice of the frame of reference, which may be noninertial. Therefore, the standard physical interpretation of the results of the theory is also possible only by the particularization of a frame of reference in which the quantum system can be regarded as nonrelativistic.

The first consistent realization of this program was given by Gorbatsevich [1]. He formulated nonrelativistic quantum mechanics in \( V_{1,3} \) for a particle of spin \( \frac{1}{2} \), generalizing the standard scheme in the configuration representation with a priori Hilbert structure of the state space. Operators of the observables (the position and momentum of the particle) are introduced by the simplest generally covariant generalizations and by the requirement of fulfillment of canonical commutation relations. To calculate the Hamiltonian, one must naturally use the generally relativistic Dirac equation. Apart from this circumstance, Gorbatsevich's approach can be characterized as inductive. In this sense, the approach adopted in the present paper is completely deductive. In Sec. 2, after a brief discussion of the concepts of a "frame of reference" and a "coordinate system," we formulate generally relativistic quantum mechanics as a basic construction independent of a frame of reference, or, more precisely, not containing such a concept explicitly. Our generally relativistic quantum mechanics is, in essence, the theory of a linear complex scalar field in \( V_{1,3} \) augmented by a definition of generally relativistic forms of operators of the observables. This is the most complicated and original part of the formulation in \( V_{1,3} \) and will be given separately in a following paper with the same title.

In Sec. 3, we obtain the \( c^{-2} \) asymptotics of the equation of motion of the generally relativistic quantum mechanics by separating appropriate classes of solutions of this equation. The separation of each class is equivalent to the introduction of a definite frame of reference, with respect to which the quantum motion of the particle can be assumed to be nonrelativistic. In Sec. 4, we show how to deduce from the generally relativistic quantum mechanics the standard scalar product and a generally covariant Schrödinger equation in \( V_{1,3} \) with Hermitian Hamiltonian containing corrections of any order in \( c^{-2} \). The conservation of the norm and the Born interpretation of the wave function are discussed in Sec. 5.

In contrast to [1], we here consider the quantum mechanics of a spinless particle. This is done not only for the sake of greater transparency of the conceptions but also because the bosonic case is of particular interest from the point of view of study of the quantum creation of particles in nonstationary external fields and, in particular, in the early universe.

The present paper merely sketches an approach to the problem of separating the nonrelativistic content of the generally relativistic theory, and it is therefore natural to limit