NONLINEAR EFFECTS IN THE CASE OF BENDING OF A WEAK D-TYPE IONIZATION FRONT

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The effect of nonlinearity on the change of shape of the surface of an unstable weak D-type ionization front is studied by the method of small perturbations. It is supposed that the gas ahead of and behind the front is incompressible. Expansion terms up to the second order inclusive are taken into account. It is shown that, in contrast with the deformation of a flame front and in the case of development of Rayleigh-Taylor instability, sections of the front with a large curvature are bulged not to the side of the less dense gas, but in the opposite direction.

In the presence of an external source of ionizing radiation, as shown by Kahn [1], the motion of a gas can be accompanied by the appearance of ionization discontinuities, characterized by a cycle of transition from a state of almost total ionization of the gas to a neutral atomic state. These discontinuities originate, in particular, when the source is located downstream and the material existing behind the discontinuity is transparent at least for part of the spectrum of the ionizing radiation, but the material ahead of the discontinuity has a powerful absorbing capability in the spectral range being considered. Under cosmic conditions, ionization discontinuities can be created by the movement of the interstellar medium relative to the sources of ionizing radiation and by star bursts.

The interest toward a study of the stability of ionization fronts is due to attempts to explain the anomalous structure of the outer parts of H II zones (the formation of "elephant trunks" and rinas) and to analyze the feasibility of the isolation of clouds of neutral gas [2-12]. In recent years, the question of the scale of the irregularities created by the movement of an ionization front has acquired importance in connection with estimates of the amplitude fluctuations of the emissions from pulsars, due to scattering of electromagnetic waves by inhomogeneities of the interstellar medium [13, 14].

Suppose that an unperturbed ionization front exists in the plane \( x = 0 \), the velocity of a neutral gas \( u_0 \) is oriented to the positive side of the \( x \) axis, and radiation is incident normally to the front (Fig. 1). Behind the discontinuity the velocity \( u_0 = \lambda u_0 \) (for weak D-type ionization fronts \( \lambda > 1 \), which follows from the relations at the discontinuity). The density \( \rho_0 \) ahead of the front is a factor \( \lambda \) greater than the density \( \rho_2 \) behind it, so that \( \rho_0 = \lambda \rho_2 \). Two-dimensional perturbations of a plane front will be considered. Just as in [6-10], the perturbed flow is assumed to be isothermal with temperature \( T_1 \) of the neutral gas and \( T_2 \) of the plasma behind the front. The density \( \rho \) and the components of the velocity \( u_1 \) and \( v_1 \) ahead of the front are related with the values of these same functions behind the front \( \rho_2, u_2, \) and \( v_2 \) by the relations [15]

\[
-p_1 v_1 (u_2 - u_1) = (2\rho_2 RT_1 - \rho_1 RT_1) \cos \varphi \\
-p_1 v_1 (v_2 - v_1) = -(2\rho_2 RT_2 - \rho_1 RT_1) \sin \varphi \\
\frac{\partial h}{\partial t} \left[ 1 + \left( \frac{\partial h}{\partial y} \right)^2 \right]^{-\nu} = u_1 \cos \varphi - v_1 \sin \varphi - v_1 \\
\frac{\partial h}{\partial t} \left[ 1 + \left( \frac{\partial h}{\partial y} \right)^2 \right]^{-\nu} = u_2 \cos \varphi - v_2 \sin \varphi - v_2, \quad \rho_1 U_1 = \rho_2 U_2 = \rho_2 u_2 \cos \varphi = \rho_2 u_2 \cos \varphi.
\]

Fig. 1


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Here $U_1$ and $U_2$ are the projections of the velocity of the gas relative to the front on the normal to it and $R = h(y, t)$ is the equation of the surface of the front.

Assuming that the amplitude $A$ of the perturbations imposed on the main flow is quite small, the solution of the problem can be presented in the form of a normal expansion in powers of $A$. We shall take into account terms up to the second order, inclusive.

Then, perturbations of velocity and density over both sides of the discontinuity are represented conveniently in the form

\[
\frac{u_1}{u_0} = 1 + Au_{11} + A^2u_{12}, \quad \frac{v_1}{u_{11}} = Av_{11} + A^2v_{12}, \quad \frac{\rho_1}{\rho_{11}} = 1 + A\rho_{11} + A^2\rho_{12},
\]

\[
\frac{u_2}{u_{21}} = 1 + Au_{21} + A^2u_{22}, \quad \frac{v_2}{u_{21}} = Av_{21} + A^2v_{22}, \quad \frac{\rho_2}{\rho_{21}} = 1 + A\rho_{21} + A^2\rho_{22}.
\]

The functions $u_{11}$, $v_{11}$, $\rho_{11}$, $u_{21}$, $v_{21}$, and $\rho_{21}$ are the solution of the problem satisfying the conditions at the discontinuity in linear approximation and have the form

\[
u_{11} = \exp(\delta t + kx) \cos hy, \quad v_{11} = -\exp(\delta t + kx) \sin hy
\]

\[
\rho_{11} = -m^2(\lambda \eta + 1) \exp(\delta t + kx) \cos hy
\]

\[
u_{21} = \frac{\gamma_1}{\eta} e^{-\delta t + kx} + \frac{\gamma_2}{(\delta/ku_{21} - 1)} \exp(\delta t - kx) \cos hy
\]

\[
\rho_{21} = \gamma_1 M_2(\delta/ku_{21} - 1) \exp(\delta t - kx) \cos hy
\]

\[
m^2 = u_{21} / RT_1 < 1, \quad M_2 = u_{22} / 2RT_1.
\]

The quantity $\eta(\lambda) = \delta / h u_{21}$ is defined by linear theory and $k$ is related with $\eta$ and $M^2$ by the relation

\[
\delta / ku_{21} = M^2 \eta(1 + M^2 \eta)^{-1} + 1 = M^2 \eta(1 + M^2 \eta)^{-1}.
\]

It follows from the conditions at the front that

\[
\gamma_1 = (\lambda \eta + 1) / (\lambda (1 - \delta / ku_{21})(1 - M^2)), \quad \gamma_2 = (\lambda \eta)^{-1}.
\]

We denote the shape of the surface of the discontinuity in the form

\[
h(y, t) = Ah^{-1}h_1 + A^2h^{-1}h_2 \quad (h_1 = \gamma_1 e^{\delta t} \cos hy, \quad h_2 = (\lambda \eta)^{-1}).
\]

Substituting $u_1, \ldots, \rho_1, u_2, \ldots, \rho_2$ in the equations of motion and continuity, and assuming that the perturbed flow is incompressible both ahead and behind the front, we obtain the following equations for determining $u_{21}, \ldots, \rho_{22}$:

\[
\frac{\partial u_{11}}{\partial x} + \frac{\partial v_{11}}{\partial y} = 0
\]

\[
\frac{\partial u_{12}}{\partial t} + u_{11} \frac{\partial u_{12}}{\partial x} + RT_1 \frac{\partial \rho_{12}}{\partial x} = - u_{21} \left( \frac{\partial u_{11}}{\partial x} + v_{11} \frac{\partial u_{11}}{\partial y} \right)
\]

\[
\frac{\partial u_{21}}{\partial t} + u_{21} \frac{\partial u_{21}}{\partial x} + RT_2 \frac{\partial u_{21}}{\partial x} = - u_{21} \left( \frac{\partial v_{11}}{\partial x} + v_{11} \frac{\partial v_{11}}{\partial y} \right)
\]

\[
\frac{\partial u_{22}}{\partial t} + u_{22} \frac{\partial u_{22}}{\partial x} + 2RT_2 \frac{\partial u_{22}}{\partial x} = - u_{21} \left( \frac{\partial u_{21}}{\partial x} + v_{21} \frac{\partial u_{21}}{\partial y} \right)
\]

\[
\frac{\partial v_{21}}{\partial t} + u_{21} \frac{\partial v_{21}}{\partial x} + 2RT_2 \frac{\partial v_{21}}{\partial x} = - u_{21} \left( \frac{\partial v_{21}}{\partial x} + v_{21} \frac{\partial v_{21}}{\partial y} \right).
\]

The boundary conditions when $x = 0$ are

\[
- \rho_0 u_{21} \left[ \frac{u_{21}}{h} \frac{\partial u_{21}}{\partial x} \xi_1 + u_{22} u_{21} - \frac{u_{21}}{h} \frac{\partial u_{21}}{\partial x} \xi_1 - u_{22} u_{21} \right] = 2RT_2 \left[ \frac{u_{21}}{h} \frac{\partial \rho_{21}}{\partial x} \xi_1 + \rho_{22} \rho_{21} \right] - RT_2 \left[ \frac{u_{21}}{h} \frac{\partial \rho_{21}}{\partial x} \xi_1 + \rho_{22} \rho_{21} \right].
\]