LAMINAR STEADY FLOW REGIMES OF TWO-COMPONENT DISPERSED SYSTEMS. VERTICAL FLOWS

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The possible flow regimes of a disperse system in vertical tubes have been studied on the basis of the hydrodynamic model of [1]. The boundary conditions are written neglecting the near-wall layer which has a thickness of the order of the average distance between particles.

It follows from the results of [1] that the effective rheological characteristics of a continuous medium consisting of a viscous fluid and particles suspended in it depend significantly not only on the physical parameters of the phases, but also on the type of motion. The peculiarities of the mechanical behavior of such media (for example, the concentration field of the dispersed phase in the flow), are completely different for flows which are oriented differently relative to the direction of action of the external mass forces, and are determined to a considerable degree by the gradients and rates of change of the flow parameters. Therefore the need arises for an independent study of the various characteristic forms of motion of disperse media.

In the following we consider the problem of steady motion in a vertical tube of constant cross section, when pulsations of the phases are the result of the specific pseudoturbulence of the disperse systems [1], and there is no conventional turbulence. Only the transverse components of the phase pulsation pressure and viscosity tensors are significant for vertical flows, and these components may be easily estimated from the results of [1].

In the pseudoturbulent flow regime, when $T_\rho$, the trial-particle time for relaxation to the conditions of motion of the disperse medium, is considerably less than the time $T$, which is the mean time between subsequent collisions of this particle with the neighbors, there is an analogy between the momentum transport by the phases of the disperse system and the transport in a turbulent homogeneous fluid, which leads to the appearance of Reynolds stresses [1]. After calculations, for the transverse components of the mean-square phase pulsation velocities $v'$ and $w'$, the pulsation pressures $P_1$, $P_2$ and the viscosities $\eta_1$, $\eta_2$, we obtain the following relations (we orient the x axis opposite the gravity force):

$$v'^2 = G_1(p, \eta, c_e, c_s) \left( \frac{P}{\beta} \right)^2,$$

$$w'^2 = G_2(p, \eta, c_e, c_s) \left( \frac{P}{\beta} \right)^2,$$

$$c_x = \frac{d \beta}{d \rho} c_e - \frac{p \beta}{\rho^2},$$

$$c_y = \frac{d \beta}{d \rho} c_s - \frac{p \beta}{\rho^2},$$

$$\eta_1 = G_3(p, \eta, \Phi(p, \rho, c_e, c_s) \eta_0, \eta_2 = G_4(p, \eta, \Phi(p, \rho, c_e, c_s) \eta_0, \eta_1,$$

$$P_1 = \rho d_1 w'^2, H_1 = \omega_0 (1 - \rho),$$

$$P_2 = \rho d_2 w'^2, H_2 = \omega_0 (1 - \rho),$$

$$\Phi = \left[ L^2 + \rho c_e \left( \frac{2}{c_x} \right) \left( 1 + \frac{c_y}{1 - \rho} \right) \right],$$

$$\eta_1 = \frac{d_1^2}{\beta} \left( \frac{P}{\beta} \right)^2, \quad \eta_2 = \frac{d_2^2}{\beta} \left( \frac{P}{\beta} \right)^2,$$

$$L = \frac{1 - \rho}{K} \left[ 1 + (1 - \rho + \frac{\partial}{\partial \rho}) \right],$$

$$\eta_1 = d_1 (1 - \rho) T \omega^2, \quad \eta_2 = d_2 T \omega^2,$$

which follow from (0.1), represent the Lagrangian scales for the correlations of the phase motion pulsations. The relations for $\eta_1$ and $\eta_2$ may be obtained from a consideration of the fluctuations in large volumes of a disperse system, containing on the average $N \gg 1$ particles, and from an analysis of the pulsations in the limit as $N \to 1$ (see [1]). It is not difficult to see that both methods yield the same relations (0.1), so that $TV$ and $TM$ determined for large volumes are proportional to $N$.

In the pseudogaseous regime, when $T_\rho >> T$ and the primary role in momentum transfer between particles is played by their indirect collisions, the expressions for $v'^2$, $w'^2$, $P_2$ and $\eta_2$ retain the previous form, but the quantities $P_1$, $P_2$ and the viscosities $\eta_1$, $\eta_2$ are determined from the analogy of the dispersed phase with a dense gas of rigid spheres. In this regime we have

$$P_2 = \rho (1 + Y(p)) d_2 w'^2, \quad \eta_2 = \rho \omega T \omega^2,$$

$$\lambda_2 = \left( \frac{12 \pi n \eta_2}{\eta_2} \right)^{-1},$$

$$T_\omega = 8 \left[ \frac{1}{Y(p)} + 0.8 - 0.76 Y(p) \right] \frac{\lambda_2}{\omega^2},$$

$$Y = \frac{(p/p_0)^{1/3}}{1 - (p/p_0)^{1/3}}, \quad n = \rho / \omega_0.$$

Here $\rho_0$ is the particle volume. Relations (0.2) are approximately valid for $\rho > 0.10-0.16$; for small $\rho$ we have $Y(p) \approx 4p$. Defining the mean free path $\lambda$ in the concentrated system as the difference between the radii of the particle specific volumes in the considered state and in the densely packed state, we obtain the estimates

$$T_\rho \approx \frac{\rho \omega}{\eta^2}, \quad T \approx \frac{\lambda}{(2^\omega + \omega^2)^{1/2}}, \quad \lambda = \rho \omega \left( \frac{1 - \rho}{\rho} \right)^{1/2},$$

$$2^\omega + \omega^2 = G_6(p) \left[ \left( \frac{L}{\rho} \right)^2 + 5 \frac{\omega^2}{\omega_0} \Phi \right] \frac{1}{\beta}.$$

Here we have used the expression for the constant $\omega_0^2$ of the longitudinal velocity pulsation, following from [1]. We see from (0.3)
that for systems which are of primary practical interest the pseudoturbulent regime is realized over nearly the entire \( \rho \) region with the exception of a very narrow interval adjacent to \( \rho = \rho_1 \). The only exceptions are suspensions of large heavy particles in a gas.

\[ 0 = (1 - \rho)P + \frac{1}{r} \frac{d}{dr} \left[ (\mu + \eta_1) r \frac{dv}{dr} \right] - d_{e}(1 - \rho)g - \beta \rho K u, \]

\[ 0 = \rho P + \frac{1}{r} \frac{d}{dr} \left[ \eta_2 r \frac{dw}{dr} \right] - c_2 \rho g + \beta \rho K u, \]

\[ P = \text{const}, \]

\[ (1 - \rho) \frac{\partial P}{\partial r} + \frac{dP_1}{dr} = \rho \frac{\partial P}{\partial r} + \frac{dP_2}{dr} = 0, \]

\[ S = 1 + \frac{5y - 2 - 7\gamma \nu_0^{3/2} - 4/3(\gamma - 4) \rho^{5/2}}{2(\gamma + 1) - \gamma \nu_0^{3/2} + (5\gamma - 2) \rho^{5/2}}. \quad (1.1) \]

It is not difficult to write similarly the equations for the motion in tubes of different cross section.

\[ M = (1 - \rho)P_2 - \rho P_1 = \text{const}. \quad (1.2) \]

In (1.1) \( \mu \) is the effective viscosity of a dispersive medium which is filtering through a porous body with fixed skeleton and the porosity \( \varepsilon = 1 - \rho \); the function \( S(\rho) \) was calculated in [2], and \( \gamma \) represents the ratio of the particle material viscosity to \( \mu_0 \). We note that \( \rho \) also includes components which are the result of both molecular motions in the liquid and its pulsations for motion in a disordered porous space.

In the limiting cases \( \mu + \eta_1 \gg \eta_2 \) or \( \eta_2 \gg \mu + \eta_1 \) and comparable gradients of the phase velocities, from (1.1) follows: \( u = u(\rho) \) and, further, from (1.2), \( \rho = \text{const} \) over the entire flow region with the possible exception of the thin near-wall layer, where, first, the gradients of \( v \) or \( w \) may be large, and second, there is degeneration of the phase pulsation velocities. It is clear that the thickness of this layer must be several times the value of \( \lambda \) from (0.3). Experiments show that in many cases even with comparable phase viscosities the particle concentration outside the near-wall layer of thickness \( l \) is approximately constant, and the phase velocity profiles are spaced nearly equidistantly (see, for example, [3]). Bearing in mind the difficulties which arise in the formulation of the boundary conditions for dispersive systems, in the following we use the assumption of constant concentration in the flow core, extending this assumption to the entire flow region. We note that this assumption is usually made not only for vertical motions, but also for horizontal as well [4]. On the whole the phase velocity profiles have the form of the solid curves in Fig. 1, which represents the situation when these profiles approach one another in the near-wall region because of the retarding effect of the wall on the particles. Replacing in the region \( l_0 \leq n \leq l \) the actual profiles by the approximate rectilinear profiles (dashed lines in Fig. 1), we obtain the boundary condition for \( n = l_0 \) in the form

\[ v_0 - w_0 = u = \text{const}, v_0, w_0 = v, w = 0, \quad (1.3) \]

The quantity \( l_0 \) represents, on the one hand, the averaged thickness of the liquid interlayers between the wall and the adjacent particles, and on the other hand it characterizes the minimal distance at which...