In recent years experiments have been conducted with very strong shock waves ($M \approx 100$). The essence of the experiments is the following (Fig. 1). At the initial instant of time a discharge takes place in the discharge chamber 1 which is filled with the test gas and which communicates with tube 2. The discharge plasma is ejected into tube 2 forming a shock wave which propagates in the gas. Ionized gas, which serves as the test medium, travels behind the shock wave.

One of the methods of studying plasma is that of the displaced magnetic flux [1, 2]. The essence of this method follows. The shock wave and the associated plasma travel in the longitudinal magnetic field created by the coil 3. The plasma has high electric conductivity; therefore as it moves it displaces the magnetic field. However, the plasma conductivity is finite, therefore the magnetic field diffuses into the plasma, and the magnetic flux, displaced by the plasma, diminishes with time. The rate of penetration of the magnetic field is recorded by the measuring coils 4, 5. From the known rate of diffusion of the magnetic field we can judge the plasma conductivity and from that we can judge the temperature, if we consider the plasma to be fully ionized. This procedure was followed basically in [1, 2].

The present study was made to evaluate the role of thermal conductivity in the process of diffusion of a magnetic field into a plasma.

First the linear problem of the diffusion of a magnetic field into a cylindrical conductor without account for thermal conductivity is solved. This problem is solved using the method of integral relations and the method of expanding the unknown functions in series, with the latter method being used to evaluate the accuracy of the first. In this case there is no assurance of uniform convergence of the method of separation of the variables since the initial conditions are not coupled with the boundary conditions. Then the question is analyzed of the cooling of a plasma moving in a tube. Estimates are made of the maximal permissible times for processes in which the cooling does not affect the plasma moving in the center of the tube. Finally, the problem of the diffusion of the magnetic field into a plasma which is cooled by the walls is solved.

All the analyses are carried out under the assumption of one-dimensionality of the processes, i.e., the diffusion and thermal layers are assumed quite thin in comparison with the characteristic linear dimension of the plasma slug. The plasma is considered as a solid conductor with the conductivity $\sigma$ and the coefficient of thermal conduction $\lambda$, calculated using the corresponding equations for a fully ionized gas.

$\text{Fig. 1}$

The method of integral relations is suggested in the paper for the calculation of the rate of diffusion of the magnetic field into a plasma which is cooled by the walls. Strictly speaking, the method is valid only for thin thermal and diffusional layers. An estimate is made of the maximal permissible time for which the plasma in the center of the tube is still essentially not affected by the cooling action of the walls. If the time of travel of the plasma to the measuring coil is greater than or equal to the maximal permissible time, this method yields only the qualitative nature of the plasma cooling with increase of the velocity.

§1. THE LINEAR PROBLEM OF MAGNETIC FIELD DIFFUSION INTO A PLASMA. METHOD OF INTEGRAL RELATIONS.

Let us consider the space between two coaxial cylinders $R_1$ and $bR_1$ ($b > 1$) and the interior of the cylinder $R_1$. Let the space $R_1 < R < bR_1$ be filled at the initial time by a uniform longitudinal magnetic field with the intensity $H_1(0)$ and let the conductivity of the medium in the interval $R_1 < R < bR_1$ equal to zero. the interior of the cylinder $R_1$ is a conductor with the conductivity $\sigma$. At the initial time there is no magnetic field inside the conductor. The cylinder $bR_1$ is a screen for the magnetic field. For all $t > 0$ the magnetic field diffuses within the conductor.

The formulated problem simulates the process of magnetic field diffusion into a conductor of finite length moving in a magnetic field under the condition that the entrance time into the uniform magnetic field is much less than the characteristic process time, and that the diffusion layer is sufficiently thin in comparison with the characteristic linear dimension of the conductor.

In the formulated one-dimensional problem the process of magnetic field diffusion into the conductor is described by the diffusion equation

$$
\frac{\partial H}{\partial t} = \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial H}{\partial R} \right), \quad \nu = \frac{\sigma^2}{4\pi \lambda} \quad (0 < R < R_1). \quad (1.1)
$$
The magnetic field in the region between the cylinders is uniform, and its intensity \( H_1(t) \) is determined in the course of the solution from the condition of constancy of the magnetic field intensity flux through the section \( bR_1 \) of the cylinder

\[
\frac{\partial}{\partial t} \int_0^{bR_1} H(R, t) 2\pi R dR = H_1(0) \quad (\pi b^2 R_1^2 - \pi R_1^2). \quad (1.2)
\]

Since \( H(R, t) = H_1(t) \) for \( R_1 < R < bR_1 \), the condition (1.2) may be rewritten in the form

\[
H_1(0) - H_1(t) = \frac{2}{(b^2 - 1) R_1^2} \int_0^{R_1} H(R, t) RdR. \quad (1.3)
\]

Switching to dimensionless variables using the formulas

\[
r = \frac{R}{R_1}, \quad \tau = \frac{v}{R_1} t, \quad \tilde{h} = \frac{H(R, t)}{H_1(0)},
\]

we obtain the following final formulation of the problem:

\[
\frac{\partial \tilde{h}}{\partial \tau} = - \beta \frac{\partial \tilde{h}}{\partial r} \quad (r > 0, 0 < r < 1),
\]

\[
h(r, 0) = 0 \quad (0 < r < 1),
\]

\[
\beta \int_0^1 hr dr - 1 - h_1(r),
\]

\[
\hat{h} = \beta = \frac{2}{b^2 - 1}. \quad (1.7)
\]

In place of the integral relation (1.7), which serves for determining \( h_1(\tau) \), we can write the differential expression

\[
\frac{\partial h_1}{\partial \tau} = - \beta \frac{\partial h_1}{\partial r} \quad (r = 1).
\]

Condition (1.8) is a corollary of Eq. (1.5), integrated with respect to \( r \) from 0 to 1, and of the integral relation (1.7).

In the experiments we measure the integral characteristics and the displaced magnetic fluxes; therefore it is advisable to make use of an approximate method which will reflect correctly only the change of the magnetic flux through the cylinder \( R_1 \). Such a method is known from boundary layer theory and is the method of integral relations.

We shall consider that the diffusion layer has the thickness \( \delta \). Outside of this layer, in the vicinity of the axis of the cylinders, the magnetic field intensity is zero. We assume a profile of the intensity \( h \) in the form

\[
h(r, \tau) = h_1(\tau) \left[ 1 - 2 \left( \frac{r}{\delta} \right) + \left( \frac{r}{\delta} \right)^2 \right]. \quad (1.9)
\]

This profile satisfies the conditions

\[
h = 0, \quad \frac{dh}{dr} = 0 \quad \text{for} \quad r = 1 - \delta,
\]

\[
h = h_1(\tau) \quad \text{for} \quad r = 1.
\]

For determining \( h_1(\tau) \) and \( \delta(\tau) \) we have from (1.7)

\[
h_1(\tau) = \frac{12}{12 + \delta (4 - \delta)}, \quad (1.10)
\]

or

\[
\delta = 2 \left[ 1 - \left( 1 + \frac{\delta}{\delta_1} - 1 \right)^{\frac{v}{R_1}} \right]. \quad (1.10)
\]

Substituting (1.10) into (1.8) we obtain the equation for \( h_1(\tau) \):

\[
\frac{\partial h_1}{\partial \tau} = \frac{\delta}{1 - \sqrt{1 - 3(1/\delta_1 - 1)\delta}}. \quad (1.11)
\]

Equation (1.11) may be integrated completely. Its integral, satisfying the zero initial condition, has the form

\[
\tau = - \beta \ln h_1 - \beta \sqrt{1 + 3\beta} \times
\]

\[
\int \ln \left( \frac{h_1 - h_1^*}{h_1 + h_1^*} \right) \frac{1 + \sqrt{1 - B}}{1 - \sqrt{1 - B}} \times
\]

\[
+ 23B (\sqrt{1 - B} - \sqrt{1 - B}) (B = \sqrt{3 + 3\beta}) \quad (1.12)
\]

The integral (1.12) has quite a complex form; therefore in the following we shall use the approximate solution of the Eqs. (1.10), (1.11).

Neglecting in (1.10) the term with \( \delta^2 \) in the denominator, we obtain

\[
h_1(\tau) = \frac{3}{\delta + \delta_1}. \quad (1.13)
\]

Substituting (1.3) and (1.11) and neglecting terms of order \( \delta^2 \) and higher, we obtain after integrating

\[
\delta = \sqrt{12 \tau}. \quad (1.14)
\]

Calculations which have been made show that the function

\[
h_1(\tau) = \frac{3}{3 + \beta \sqrt{12 \tau}} \quad (1.15)
\]

well approximates the function \( h_1(\tau) \) defined by the integral (1.12).

Let us write the expression for the magnetic flux \( \Phi \) displaced by the conductor. We denote by \( H_1^\infty \) the intensity of the undisturbed magnetic field ahead of the conductor which travels along the tube. The intensity flux through the coil of radius \( R_1 r_1 \) is \( \pi r_1^2 R_1^2 H_1^\infty \).

At some time \( t \), the flux through the coil in the disturbed magnetic field, becomes equal to

\[
2\pi \int_o^{R_1} H R dR + \frac{2\pi (r_1^2 - 1)}{2} R_1^2 H_1(\tau). \quad (1.16)
\]

The flux change, registered by the coil, is

\[
\Phi = \pi R_1^2 r_1^2 H_1^\infty - 2\pi \int_o^{R_1} H R dR - \pi R_1^2 (r_1^2 - 1) H_1(\tau).
\]