SOME ASPECTS OF CALCULATING SUBSONIC FLOW OVER WINGS OF COMPLEX PLANFORM

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The use of wings of complex planform is characteristic of the present stage of development of aviation; with discontinuities along the leading and trailing edges; with curved edges; with variable geometry (by pivoting the wing panels). This article considers some aspects of the calculation of the over-all and distributed aerodynamic characteristics of such wings for low and high subsonic velocities. The methods, based on the lifting surface scheme and the use of discrete vortical singularities, enable quite efficient and reliable digital computation of the flow about these wings at moderate angles of attack. For steady motion of the wing a further development of the method of [1] is obtained, for harmonic oscillations an extension of [2] is obtained, and for aperiodic motions of the wing and gust inputs a modification of the method of [3] is found.

1. Wing vortex scheme and notation system. Let us consider a monoplane wing — a flat plate of arbitrary symmetric planform. We introduce the xyz coordinate system fixed with the wing, locating the coordinate origin at the leading edge at the root section (Fig. 1). We direct the x-axis downstream along the root chord b, the y-axis spanwise to the right, the y-axis perpendicular to the plane of the wing.

In the general case, the leading and trailing edges of the wing may be curvilinear and have discontinuities. We denote the wing semispan by 1/2, the dimensions of the segments between neighboring discontinuities along z will be l/2, l/2, ..., l/2 (i is the total number of discontinuities, considering that there is always a discontinuity at the root section); then

\[ \frac{l}{2} = \sum_{i=1}^{i} \frac{l_i}{2}. \]  

(1.1)

We shall replace the wing by a vortex surface, consisting of bound and free vortices (the latter are also located behind the wing). As done previously [1-3], the continuous vortex layer is replaced by a discrete system of skewed horseshoe vortices. At the sections \( z = \text{const} \), where there are discontinuities, the circulation of the transverse filament may have discontinuities, therefore the ends of the corresponding oblique vortices must be located at the chords passing through the breakpoints.

We divide each segment \( l_i/2 \) into \( N_e \) equal parts, and we obtain systems of strips of width \( \Delta l_e \) (\( e = 1, 2, \ldots, i \)) (Fig. 1). The edges of the strips are numbered from right to left (\( k = 0 \) coincides with the wingtip, \( k = N \) coincides with the root chord). Let the sections passing through the breakpoints have the numbers \( k = N_1, k = N_2, \ldots, k = N_i \); then

\[ \frac{\Delta l_e}{b} = \frac{l_i}{2N_e}, \quad N_e = N_e - N_{e-1}, \]

\[ N_i = N, \quad N_0 = 0 (e = t, 2, \ldots, i). \]

The selection of the numbers \( N_e \) is made on the basis of experience accumulated in calculations and is associated primarily with the wing planform.

We shall characterize the transverse vortex lines on the wing by the number \( \mu \), counting them from the leading edge and including in this system the leading and trailing edges (\( \mu = 0 \) is the leading edge, \( \mu = n + 1 \) is the trailing edge, \( n \) is the number of vortex filaments in each section, \( z = \text{const} \)). We shall denote the coordinates of the points located at the intersection of the lines \( \mu \) and \( k \) by the subscripts \( \mu \) and \( k \). We consider the coordinates of the points of the leading and trailing edges,

\[ x_{0\mu}/b, \quad x_{n+1\mu}/b, \quad z_{0\mu}/b = z_{n+1\mu}/b, \]  

(1.3)

known (the wing planform is given).

In selecting the locations of the vortices and the computational points, which are denoted by small crosses in Fig. 1, we make use of the reasoning of [1]. Each chord \( b_k \) of the section \( k \) is divided into \( n \) equal segments, and each of these is further divided into four equal parts. The points lying at a distance of 1/4 of the distance from the segment upper edge are the ends of the transverse vortex. The computational points at which the boundary conditions are satisfied lie at the midpoint between the lines \( k \) and \( k - 1 \), and also the midpoint between the lines \( \mu \) and \( \mu - 1 \). We shall specify their position by the numbers (Fig. 1) of the transverse lines \( \nu \) and the longitudinal lines \( p \), where \( 1 \leq \nu \leq n, \quad 1 \leq p \leq N \).

Thus, the wing is replaced approximately by 2N trapezoids with bases parallel to x and heights \( \Delta l_e \).

2. Basic geometric relationships. Assuming quantities (1.3) to be given, we determine the geometric parameters of the wing, the coordinates of the transverse vortices and of the computational points. As the characteristic linear dimension we shall always use the root chord \( b \).

For the variable chord of the section \( k \) we have

\[ b_k/b = x_{n+1k}/b - x_{0k}/b \]  

(2.1)

We find the wing area, then the aspect ratio and the taper ratio,

\[ S = \sum_{i=1}^{i} \int_{x_{0i}}^{x_{n+1i}} b_i \, dx, \quad \lambda = \frac{l}{b}, \quad \eta = \frac{b}{b_0}. \]  

(2.2)

Here \( b_0 \) is the tip chord. On the basis of the above discussion, for the coordinates \( x_{\mu k}, \quad z_{\mu k} \) of the point of origin of the transverse vortex we can write

\[ x_{\mu k} = x_{0k} + \frac{b_k}{\eta} + (\mu - 1) \frac{b_k}{\eta}, \quad z_{\mu k} = z_{0k}. \]  

(2.3)
Hence,
\[
\frac{x_{pk}}{b} = \frac{x_{nk}}{b} + \frac{b}{b} \cdot \frac{\mu - \frac{n}{n}}{\frac{n}{n}}, \quad \frac{z_{pk}}{b} = \frac{z_{nk}}{b} \quad (1 \leq \mu \leq n, \quad 0 \leq k \leq N). \tag{2.4}
\]

We denote the coordinates of the midpoint of the transverse vortex segment connecting the points \(\mu, k\) and \(\mu, k - 1\) by \(x_{\mu k}^{\mu k-1}\) and \(z_{\mu k}^{\mu k-1}\), and
\[
\frac{x_{\mu k}^{\mu k-1}}{b} = \frac{1}{2} \left( \frac{x_{nk}}{b} + \frac{x_{n k}}{b} \right), \quad \frac{z_{\mu k}^{\mu k-1}}{b} = \frac{1}{2} \left( \frac{z_{nk}}{b} + \frac{z_{n k}}{b} \right) \quad (1 \leq \mu \leq n). \tag{2.5}
\]

We denote the span of this segment by \(l_{k, k-1}\) (it is independent of \(\mu\)); then
\[
\frac{l_{k, k-1}}{b} = \begin{cases} 
\Delta l_j/b \text{ for } k \leq N_1, \\
\Delta l_j/b \text{ for } N_1 < k < N_\delta, \\
\Delta l_j/b \text{ for } N_\delta < k < N_1.
\end{cases} \tag{2.6}
\]

The sweep angle of this vortex is
\[
tg \chi_{\mu k}^{\mu k-1} = \left( \frac{x_{\mu k}^{\mu k-1} - x_{\mu k}}{b} \right) \frac{b}{l_{k, k-1}} \quad (1 \leq \mu \leq n). \tag{2.7}
\]

The vortex which is symmetrical to that considered with respect to the xy plane and located on the left half of the wing is characterized by the following quantities:
\[
\frac{x_{\mu k}^{\mu k-1}}{b} = \frac{x_{\mu k}^{\mu k-1}}{b}, \quad \frac{z_{\mu k}^{\mu k-1}}{b} = \frac{z_{\mu k}^{\mu k-1}}{b}, \quad \frac{l_{k, k-1}}{b} = \frac{l_{k, k-1}}{b}, \quad \chi_{\mu k}^{\mu k-1} = -\chi_{\mu k}^{\mu k-1}. \tag{2.8}
\]

The quantities appearing on the right side of (2.9) are determined using Eqs. (2.4) and (2.5), in which \(\mu\) and \(k\) are to be replaced by \(\nu\) and \(p\).

3. The problem of steady translational motion of an unyawed wing. As an example let us consider in more detail the peculiarities of the solution of this problem. The velocity field in the plane of the wing resulting from a stationary oblique horseshoe vortex was found in [1] for a coordinate system with origin at the midpoint of the transverse vortex and a direction of the z axis differing from that used here. In contrast with [1], we take as the characteristic dimension the chord \(b\) rather than the span of the vortex; let \(\alpha\) be the angle of attack; for the circulation of the vortex we have
\[
\Gamma_{\mu k}^{\mu k-1} = U_0 \Gamma_{\mu k}^{\mu k-1} \alpha. \tag{3.1}
\]

For the velocity arising at the computational point with the coordinates \((x_{\mu k}^{\mu k-1}, z_{\mu k}^{\mu k-1})\) due to the oblique horseshoe vortex \((\mu, k) - (\mu, k - 1)\) (including the free vortices), we obtain
\[
\begin{align*}
\frac{W_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\Gamma_{\nu, \mu k}^{\nu, \mu k-1}}{2\pi} \left( \frac{\nu \mu k}{\nu} \right) \frac{b}{b_{\nu k}}, \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= w_{\nu, \mu k}^{\nu, \mu k-1} \left( \frac{x_{\mu k}^{\nu k} - x_{\mu k}}{b} \right), \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\nu \mu k}{\nu} \left( \frac{b_{\nu k}^{\nu k} - b_{\nu k}}{b} \right), \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\nu \mu k}{\nu} \left( \frac{b_{\nu k}^{\nu k} - b_{\nu k}}{b} \right). \tag{3.2}
\end{align*}
\]

The expressions for the functions \(w_{xy}(\xi_0, \xi_0, \chi)\) are given in [1].

The velocity induced at this same point by the vortex which is symmetric to the xy plane, according to (2.8), may be represented in the form
\[
\begin{align*}
\frac{W_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\Gamma_{\nu, \mu k}^{\nu, \mu k-1}}{2\pi} \left( \frac{\nu \mu k}{\nu} \right) \frac{b}{b_{\nu k}}, \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= w_{\nu, \mu k}^{\nu, \mu k-1} \left( \frac{x_{\mu k}^{\nu k} - x_{\mu k}}{b} \right), \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\nu \mu k}{\nu} \left( \frac{b_{\nu k}^{\nu k} - b_{\nu k}}{b} \right), \\
\frac{w_{\nu, \mu k}^{\nu, \mu k-1}}{U_0} &= \frac{\nu \mu k}{\nu} \left( \frac{b_{\nu k}^{\nu k} - b_{\nu k}}{b} \right). \tag{3.3}
\end{align*}
\]