Multiple Scattering in Random Media
II. The General 2BA for the Uncorrelated System

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The paper is an application of a general microscopic approach to the theory of the average scattering matrix for a particle interacting with random scatterers. We present a detailed treatment for the case of uncorrelated positions of the scatterers. First, the general two-body additive approximation is used to truncate the hierarchy of correlation functions for fluctuations. It is shown that the self-energy is accurate through the fourth power of the individual scattering amplitude. Second, the hierarchy is terminated at the next stage. The self-energy is correct to the sixth power of the scattering amplitude.

KEY WORDS: Multiple scattering; random media.

1. INTRODUCTION

We summarize the basic equations of the microscopic approach to multiple scattering. In the notation of I, with 2 standing for the wave vector \( k_2 \), and with a matrix notation in wave vector space, the self-energy corresponding to the ensemble averaged \( T \) matrix is given by

\[ \Sigma(2) = N\langle 2|(1 - \overline{K})^{-1}t|2\rangle \]  

The kernel is \( \overline{K} = \overline{K}_0 + \overline{K}_1 \). \( \overline{K}_0 \) is the quasicrystalline approximation part

\[ \langle 1|\overline{K}_0(2)|3\rangle = \langle 1|G_0(2)|3\rangle F_2(2 - 3) \]

where \( F_2(2 - 3) \) is the Fourier transform of the static pair distributions, and

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$K_1$ is a kernel that takes account of the fluctuations. It is given by

$$\langle 1\mid K_1(2)\mid 3 \rangle = \frac{\langle 1\mid tG_0|2 - \lambda \rangle}{N} \Delta(\lambda|0) \sum_\beta \delta E_\beta(\lambda) \langle 2 - \lambda|\Gamma_\beta|3 \rangle \quad (3)$$

Here

$$E_\beta^1(\lambda) = \sum_{\gamma \neq \beta} E_{\gamma\beta}(\lambda) = \sum_{\gamma \neq \beta} \exp\left[i\lambda(R_\beta - R_\gamma)\right]$$

and the $R_\beta$ are the site positions. This is an ensemble average involving a microscopic fluctuation $\Gamma_\alpha$. The manipulation of the fluctuation equation to exhibit collective effects gave

$$\langle 1\mid \Gamma_\alpha(2)\mid 3 \rangle = \langle 1\mid \delta j_\alpha(2)\mid 3 \rangle + \delta \sum_{\beta \neq \alpha} \langle 1\mid \delta L_{\alpha\beta}\Gamma_\beta|3 \rangle \quad (4)$$

The source term $\delta j_\alpha$ and the matrix $\delta L_{\alpha\beta}$ are given in terms of

$$\langle 1\mid (K_0)_{a\alpha}\mid 3 \rangle = N\langle 1\mid tG_0|3 \rangle E_{a\beta}(2 - 3)(1 - \delta_{a,\beta}) \quad (5)$$

The source term $\delta j_\alpha$ is

$$\delta j_\alpha = \frac{\delta K_\alpha^0}{N} + \frac{1}{N^2} \frac{K_0}{1 - K_0} \sum_\gamma \delta K_\gamma^0 \quad (6)$$

The first term is a direct term of order $t$ with the functional form of the restricted 2BA. The second term is of order $t^2$ and is a collective term involving pairs of particles other than $\alpha$.

The kernel $\delta L_{a\beta}$ is

$$\delta L_{a\beta} = \frac{1}{N} (\delta K_0)_{a\beta} + \frac{1}{N^2} \frac{K_0}{1 - K_0} \delta K_\beta^1 \quad (7)$$

Again the direct term is of order $t$ and the collective term is of order $t^2$. In the restricted 2BA we limited ourselves to an ansatz for $\Gamma_\alpha$ of the same form, replacing the direct source term $\langle 1\mid tG_0|3 \rangle$ by a quantity $H$. $H$ was determined by using a hierarchy equation to treat nonlinear fluctuations. For the uncorrelated case and for a one-dimensional $\delta$ function we found

$$\langle 1\mid H|3 \rangle = tG_0^\delta(3), \quad G_0^\delta(3) = G_0(3)/\left[1 - NtG_0(3)\right]$$

We now want to improve on this result. To this end we first provide motivation for the general 2BA. Thus an expression for $\Gamma_\alpha$ accurate to order $t^2$ is

$$\Gamma_\alpha = \delta j_\alpha + \frac{1}{N^2} \delta \sum_{\beta \neq \alpha} (\delta K_0)_{a\beta} \sum_{\gamma \neq \beta} (\delta K_0)_{\beta\gamma}$$

Let us examine the functional form of the second term. It has three distinct types of contributions. The first type comes from particle reduction $\gamma = \alpha$. 