We consider electrogasdynamic (EGD) flow in the one-dimensional approximation. The conditions for transition through the speed of sound in EGD are formulated, and its analytic expression is given. A comparison is made of experimental and theoretical data on the parameter distribution and efficiency in EGD processes.

**NOTATION**

- \( p \) is the pressure; \( \rho \) is the density; \( V \) is the velocity; \( E \) is the electric field intensity; \( \varepsilon \) is the dielectric constant; \( U \) is the potential; \( L \) is the characteristic length; \( k \) is the adiabatic exponent; \( M \) is the Mach number; \( i \) is the charge carrier index; \( 0 \) is the inlet; \( 1 \) is the exit.

Flow in which inertial and electrostatic forces predominate will be termed EGD flow. The general system of equations and boundary conditions describing the EGD problem [1] in the one-dimensional approximation for dimensionless parameters is

\[
\frac{d(pV)}{dx} = 0, \quad \rho V \frac{dV}{dx} + \frac{4}{kM_0^2} \frac{dp}{dx} - \pi_0 iE = 0, \\
\frac{k}{k-1} \frac{d(pV)}{dx} + \frac{d}{dx} \left( \frac{pV^2}{2} \right) - \pi_0 \pi_0 iV_i E = 0, \\
\frac{d(pV_i)}{dx} = 0, \quad \frac{(1 - \pi_0)}{\rho} E = V - \pi_0 V_i, \quad \frac{dE}{dx} = \pi_0 i, \\
p = \rho = V = \rho_i = E = 1 \quad \text{for} \quad x = 0. \quad (1)
\]

Here

\[\pi_e = \frac{\rho_0 \varepsilon_0 L}{\rho_0 V^2}, \quad \pi_s = \frac{\rho_0 L}{\varepsilon E_0}, \quad \pi_0 = \frac{V_0}{V}\]

are the EGD criteria characterizing the problem.

In this approximation the heat conduction and diffusion processes are not considered in (1).

Consider the first integral of system (1)

\[V - 1 + \frac{1}{kM_0^2} (p - 1) - \pi_e \left( \frac{E^2 - 1}{2} \right) = 0. \quad (2)
\]

In the case \( E^2 > 1 \), which we hereafter term the pump case, there is an increase of either the medium velocity or pressure and expenditure of electrical energy; in the case \( E^2 < 1 \) (generator regime) there is conversion of flow energy into electrical energy.

From (2) we obtain the expression for the derivative of the flow velocity along the \( x \)-axis:

\[\frac{dV}{dx} = \frac{M_0^2}{M_0^2 - 1} \pi_0 iE \left( 1 - \frac{k}{k-1} \frac{1 - \pi_0 iV_i}{V} \right). \quad (3)
\]

We see from (3) that in the generator regime \( \pi_0 < 0, \ \pi_\beta < 1, \ V > V_i \) in subsonic flow \( (M < 1) \) the flow velocity increases along the \( x \)-axis \( (dV/dx > 0) \). This conclusion is a particular case of the general proposition of gasdynamics, which implies that a subsonic flow is accelerated when work is performed.

In the pump regime \( \pi_\beta > 0, \ \pi_\beta > 1 \) we must consider two cases:

\[V_i / V < \pi_0 (k - 1) / k, \quad V_i / V > \pi_0 (k - 1) / k.
\]

In the first case the subsonic flow velocity decreases \( (dV/dx < 0) \); in the second it increases \( (dV/dx > 0) \) along the channel length. The variation of the flow characteristics in these cases is associated with the ratio of the temperature and the efficiency, i.e., with variation of the efficiency. The efficiency depends in turn on the criterion \( \pi_\beta \), i.e., on the ratio of the dimensional velocities \( V^* \) and \( V_\beta^* \).

In supersonic flow \( (M > 1) \) in the generator regime the velocity diminishes \( (dV/dx < 0) \). In the pump regime the velocity variation is determined by the ratio \( V^* \) and \( V_\beta^* \).

In gasdynamics we usually study not simply the flow velocity variation, but the mutual variation of the velocity and Mach number. From (1) we can obtain an expression relating the derivatives of the velocity and Mach number along the flow axis.

\[\frac{dM}{dx} = \frac{M}{2V} \left( 1 + kM^2 \right) \frac{dV}{dx} - kM_0 \pi_0 iE \right). \quad (4)
\]

A joint analysis of (3) and (4) is shown in Fig. 1. If \( V^* \) and \( M \) are the values of the velocity and Mach number at a given section, then in the regions \( A \) \( D \) \( E \) \( 2 \) for motion along the \( x \)-axis the point \( (V^*, M) \) moves to the right and upward; in the regions \( B \) \( C \) it moves to the left and downward; and in the regions \( D \) \( 2 \) \( E \) \( 1 \) it moves to the left and upward (in the figure these directions are shown by the arrows). We see from the figure and (3) and (4) that transition through the speed of sound \( (M = 1) \) at points differing from \( V^* = V_\beta^* \) and \( V^* = V_\beta^* \) is impossible.

Transition through the speed of sound at the point \( V^* = V_\beta^* \) means transition from the generator to the pump regime or vice versa. The device in which a smooth transition through the speed of sound is ac-
complished by means of a change of the electrical field regime may be termed an EGD nozzle. The condition for transition through the speed of sound in an EGD flow, written in the form (3), (4), is a particular case of the general equation of action reversal, formulated by Vulis [2].

Transition through the speed of sound at the point \(V^* = (k - 1)k^{-1}V^*\) is associated with a change of the internal characteristics of the regime (and not with a change of the regime itself). The analog in gasdynamics is the thermal nozzle.

Now let us consider the parameter distribution along the flow axis and the efficiency of EGD energy conversion (for an incompressible fluid).

In this case (1) admits the following analytic solution:

\[
E = \frac{1}{1 - \pi_\beta} \left( 1 - \pi_\beta \left[ 1 - \frac{2(1 - \pi_\beta)}{\pi_\beta} \pi_{\alpha x} \right]^{1/3} \right), \quad (5)
\]

\[
\rho = \frac{\pi_\beta \pi_{\alpha x}}{\pi_\alpha (1 - \pi_\beta)} \left[ 1 - \left\{ 1 - 2\pi_{\alpha x} \frac{1 - \pi_\beta}{\pi_\beta} \right\}^{1/3} \right] - \frac{\pi_\beta \pi_{\alpha x}}{1 - \pi_\beta} x + 1, \quad (6)
\]

\[
U = \pm \frac{x}{1 - \pi_\phi \pm \frac{\pi_\phi^2}{3\pi_\alpha (1 - \pi_\beta)^2} \left\{ \left[ 1 - \frac{2(1 - \pi_\beta)}{\pi_\beta} \pi_{\alpha x} \right]^{1/3} - 1 \right\}}, \quad (7)
\]

The pressure distribution along the x-axis for the selected criteria \(\pi_\beta, \pi_\phi, \text{ and } \pi_e\) is shown in Fig. 2. The possibility of the realization of two flow regimes, generator and pump, show up particularly clearly here. In the pure pump regime the external electric field in the region of charge motion supplies additional energy to the charges. A portion of this electric field energy is transferred to the neutrals by the mechanism of charge-neutral interaction. As a result there is an increase in the energy of the neutral medium, and the pressure increases along the x-axis. The conditions \(\pi_\beta > 1\) and \(\pi_e > 0\) are characteristic for the pump regime.

In the pure generator regime the external electric field retards charge motion. In this case the energy of the neutral medium is transferred by the interaction mechanism to the charged component, and later changes into energy of the electric field.

As a rule, in the generator regime the electric field intensity decreases along the channel length, while the potential increases (Fig. 3). The reverse pattern is characteristic for the pump regime.

The boundary regime is the condition \(\pi_\beta = 0\). Physically this case corresponds to the situation when the charge carrier velocity, acquired in the electric field, is equal in magnitude and opposite in direction to the neutral transport velocity, charge transfer does not take place. The electric field in the motion region is equivalent to the field of a flat capacitor with the given charge intensity.

The other limiting case is the regime \(\pi_e = 1\). In this case the mobility of the charge carriers equals zero; the charges are essentially frozen into a neutral medium.

Between the generator and pump regimes lies the region of transitional or mixed flows. We see from Fig. 3 that the intensity decreases along the x-axis in the generator regime more sharply for large criteria \(\pi_\beta\). For a certain \(\pi_\beta\) the electric field intensity at the point \(x = 1\) becomes zero. It is this range of variation of \(\pi_\beta\) which is characteristic for the pure generator regime. With further increase of \(\pi_\beta\) the field intensity changes sign as we move along the x-axis, the potential distribution curve becomes convex, and in the flow region there appears a zone where reverse conversion of the electric energy into the motion energy of the neutral stream takes place (the pressure increases). Such a flow regime may be termed a pseudogenerator regime. The limiting case of such a regime is the case \(\pi_\beta = 1\), i.e., absence of voltage generation. We note that in this case the stream pressure (Fig. 2) still diminishes, which is explained by the energy dissipation by friction of the components.

The ratio of the dissipative friction energy and the electric field energy characterizes the efficiency of the EGD process. Introducing the concept of internal efficiency for the generator and pump processes,

\[
\eta = \frac{(U_{i}^* - U_o^*)\pi_{\alpha x} V_{i}^*}{(p_{i}^* - p_o^*) V_{i}^*},
\]

we can obtain the following expressions from (5)-(8):

\[
\eta = \frac{\pi_{\alpha x}(1 - \pi_\beta)^{1/3} \pi_\beta}{1 - (1 - 2\pi_\beta)^{1/3}} + \frac{\pi_{\beta} \pi_{\alpha x}(1 - \pi_\beta)^{1/3}}{1 - (1 - 2\pi_\beta)^{1/3}}, \quad (9)
\]

\[
\eta = \frac{\pi_{\alpha x}(1 - \pi_\beta)^{1/3} \pi_\beta}{1 - (1 - 2\pi_\beta)^{1/3}} + \frac{\pi_{\beta} \pi_{\alpha x}(1 - \pi_\beta)^{1/3}}{1 - (1 - 2\pi_\beta)^{1/3}} \frac{1 - (1 - 2\pi_\beta)^{1/3}}{\pi_{\beta} (1 - \pi_\beta)^{1/3}}, \quad (10)
\]

Expressions (9) and (10) imply that the efficiency of hydrodynamic converters depends only on two criteria: \(\pi_\beta\) and \(\pi_e\).