INVESTIGATION OF THE ONE-DIMENSIONAL MOTION OF A SNOW AVALANCHE ALONG A FLAT SLOPE

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In view of the developing construction in mountainous regions where there is danger from avalanches, the problem of protection against avalanches has become timely. In the solution of this problem, various methods can be used in practice; in connection with the use of these methods, there arise a great number of engineering and mechanical problems. In particular, in the design of structures for protection against avalanches, information is needed on the parameters of moving avalanches, i.e., on velocities, heights of the front, densities of the snow, etc., that is to say, calculations of the motion of avalanches along a slope, as well as of their interaction with the structure under consideration. From a practical point of view, information on the maximal range of the throw, i.e., on the boundary of the avalanche danger zone, is also important. The present article is devoted to an analytical and numerical investigation of the one-dimensional motion of an avalanche; an asymptotic solution is obtained to the problem of the one-dimensional motion of an avalanche along a homogeneous slope.

1. The motion of an avalanche is modelled here in the same way as in [1, 2]. It is assumed that, ahead of the avalanche, there lies snow at rest (Fig. 1); the density of the snow, and the strength and thickness of the layer which peels off with the descent of the avalanche are known. At the leading front of the avalanche, this snow breaks up into lumps, which infuse into the body of the avalanche and are carried downward, mixing chaotically. This flow of snow is regarded as a turbulent flow of liquid with complex properties, and is approximately described by the following system of equations:

\[
\begin{align*}
\frac{\partial h}{\partial t} + \frac{\partial hv}{\partial s} &= 0 \\
\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial s} &= f - \frac{g}{2h} \frac{\partial h}{\partial s} \cos \psi - k \frac{|v|}{h}.
\end{align*}
\]

Here \( v \) is the mean velocity along the slope averaged over a transverse cross section of the avalanche; \( h \) is the thickness of the layer of snow, reckoned perpendicular to the slope, from the surface along which the avalanche is moving; \( t \) is the time; \( s \) is a coordinate along the slope; \( g \) is the acceleration due to gravity; \( \psi \) is the angle of inclination of the slope to the horizontal; \( k \) is the coefficient of "hydraulic" friction; \( f \) is connected with that part of the forces acting on the snow which does not depend on the absolute value of the velocity, i.e., with the force of gravity and, possibly, with the friction. In [3] for example, it is assumed that

\[
f = g \sin \psi - \mu (\text{sign } v) \cos \psi
\]

where \( \mu \) is the coefficient of "Coulomb" friction.

Equations (1.1) are written for a wide ravine or slope, under the condition that effects connected with an interaction with the walls of the ravine or with the air at the side boundaries of the ravine can be neglected.


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Let $w$ be the velocity of the leading front of the avalanche, i.e., of the boundary separating the avalanche from the snow which has not yet been brought into motion; $\rho$ and $\rho_0$ the densities of the snow in the avalanche and in the undisturbed snow covering, respectively; $h_0$ the thickness of the layer of snow covering which is peeled off by the avalanche; $\sigma^*$ the critical stress at which the snow covering breaks down.

The conditions for the conservation of mass and momentum must be satisfied at the leading front of the avalanche

$$\rho h (w-v) = \rho_0 h_0 w$$

$$\rho_0 h_0 w = \frac{1}{2} \rho g h^3 \cos \varphi - \sigma^* h_0$$

If, at the leading front

$$\frac{1}{2} \rho g h^3 \cos \varphi - \sigma^* h_0 < 0$$

the front does not move, i.e., $w = 0$ and $v = 0$. In what follows, it is postulated that $\rho = \rho_0$.

At the rear front of the avalanche (at the breakaway line) we impose the boundary conditions

$$v = 0, \quad h = 0$$

Further, as in [1, 2] it is assumed that at all points of the slope $\alpha = \sigma^*/(\rho g h \cos \varphi) > 0.5$; the extreme value $\alpha = 0.5$ corresponds to the complete breakdown of the structure of the snow covering ahead of the avalanche when the snow in front of the avalanche behaves like a liquid.

2. Let us consider the motion of an avalanche over a long flat ($\phi = \text{const}$) slope with a homogeneous ($k, h_0, \rho_0, \mu, \sigma^* = \text{const}$) snow covering with $\tan \phi > \mu$. In the case $\tan \phi < \mu$, there is drag and the avalanche stops. We postulate that the velocity of the avalanche at any point is positive and that therefore $f = g (\sin \psi - \mu \cos \psi) = \text{const}$.

We introduce the dimensionless variables

$$T = t / \sqrt{gh_0 \cos \psi}, \quad S = s / (gh_0 \cos \psi), \quad H = h / h_0, \quad V = v / \sqrt{gh_0 \cos \psi}, \quad W = w / \sqrt{gh_0 \cos \psi}$$

Equations (1.1) and the boundary conditions at the leading front (1.3) assume the form

$$\frac{\partial H}{\partial T} + \frac{\partial HV}{\partial S} = 0, \quad \frac{\partial V}{\partial T} + V \frac{\partial V}{\partial S} + \frac{\partial H}{\partial S} = 1 - \frac{V^2}{H}$$

$$H (W-V) = W, \quad WV = 0.5 H^2 - \alpha$$

The parameters which determine the conditions of the motion enter into the system of equations (2.1), (2.2) only through the two dimensionless combinations $\alpha$ and $\beta$.

We consider the case $\beta = k (\tan \psi - \mu)^{-1} > 0$ in accordance with the assumption that $\tan \psi > \mu$. We undertake the construction of an asymptotic solution of the problem (1.4)-(2.2) under the assumption that the velocity of the leading front tends toward some constant limiting velocity $W$, and that the distribution of the

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**Fig. 1**

Let $w$ be the velocity of the leading front of the avalanche, i.e., of the boundary separating the avalanche from the snow which has not yet been brought into motion; $\rho$ and $\rho_0$ the densities of the snow in the avalanche and in the undisturbed snow covering, respectively; $h_0$ the thickness of the layer of snow covering which is peeled off by the avalanche; $\sigma^*$ the critical stress at which the snow covering breaks down.

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