ELASTICOPLASTIC CONDITIONS OF THE FILTRATION
OF A LIQUID IN POROUS MEDIA

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The article discusses questions in the theory of filtration in porous media, taking account
of elastic, elasticoplastic, and plastic deformations. Parameters are introduced to evalu-
ate irreversible effects in petroleum- and water-bearing strata, i.e., coefficients of the
change in the porosity and the permeability. Equations are derived for filtration under
unsteady-state and steady-state working conditions of wells and galleries. Two limiting
cases, which allow analytical solutions, are separated out. In the general case, the equa-
tions of elasticoplastic filtration conditions are solved on an electronic computer. The nu-
merial calculations show that the predominating effect results from taking account of the
irreversible change in the permeability, depending on the change of the pressure in the
stratum.

1. The publications [1, 2] were obviously the first to draw attention to the fact that, with a lowering
of the stratum pressure, there is an irreversible deformation of the rocks, leading to irreversible changes
in the porosity and permeability of a petroleum- and water-bearing stratum. In [3], the process of filtra-
tion under such conditions was called elasticoplastic, and a simplified calculating scheme was proposed
under the assumption that only the porosity of the stratum changes in an irreversible manner as a function
of a change in the stratum pressure. Under these circumstances, the dependence of the porosity on the
pressure was assumed to be linear, both with a decrease in the stratum pressure and with its regeneration.

Laboratory and production experiments, a review of which is given in [4-6], showed that the perme-
ability and the porosity of a stratum vary irreversibly as a function of the pressure; here, the change in
the permeability is the more considerable.

Figure 1 gives a schematic representation of the character of the change in the porosity and the per-
meability as a function of the pressure in an element of the stratum (p0 and p are the initial and instanta-
nce pressures). With an initial lowering of the pressure, there is an increase in the additional load on the element of
the stratum (p0 - p), which leads to deformation of the element of the stratum. With a subsequent increase in the stratum pressure
(Fig. 1, point i, stratum pressure p1), the element of the stratum
is unloaded, with which the original values of the porosity m0 and
the permeability k0 in an element of an elasticoplastic stratum are
not completely regenerated, and reach the values m0i and k0i. The
greater the loads with which the regeneration of the pressure takes place,
the greater is the partially irreversible change in the po-
rosity and permeability and, in the limit, there can exist a case
when the deformations are completely residual, which leads to
completely or partially irreversible changes in the porosity and
permeability. There is a class of petroleum- and water-bearing
rocks for which any kind of deformations of the rocks lead to com-

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pletely irreversible changes in the porosity and the permeability. Such rocks, for example, include weakly cemented sands which have clayey materials as the cement, sands, and any kinds of rocks found in super-deep strata.

We note that, in the general case, the mechanism of the deformation of the rocks in an element of a stratum (a sample) differs from the deformation process in the stratum. In the latter case, nonlocal effects start to play a significant role; these effects were investigated in detail in [6]. Here, for simplicity, we assume that the deformation process in a sample of rock takes place in exactly the same way as in the stratum as a whole.

2. Below, we obtain the relationships for the parameters needed in the derivation of the equations and calculating formulas. We note that the present article does not make an analysis of the distribution of the stresses and the deformations which they bring about. In this connection, published work can be cited [6–8].

In general form, the character of the change in the porosity $m$ and the permeability $k$ as a function of the pressure in an element of a stratum is shown on Fig. 1. It can be seen from this that, with an initial lowering of the stratum pressure (with a load on an element of the stratum), we have a single smooth curve of the dependence of the parameters of the stratum on the pressure. We write the dependences of the porosity and the permeability on the pressure in the liquid in the form [6, 8]

$$
m = m_0 \exp [a_m(p - p_0)], 
$$

$$
k = k_0 \exp [a_k(p - p_0)], 
$$

Here $a_{m0}$ is the compressibility coefficient of the pores; $a_{k0}$ is the coefficient of the change in the permeability.

In an analysis of (2.1) it is essential to note that, with an initial decrease in the stratum pressure, the coefficients $a_{m0}$ and $a_{k0}$ are constant quantities.

With the regeneration of the stratum pressure, with any given value of the pressure (Fig. 1), the process of the change in the porosity and the permeability is described by exactly the same dependences (2.1) with the sole difference that the coefficients $m$ and $k$, $a_m$ and $a_k$ are, in turn, functions of that minimal pressure $p$ from which its regeneration started. Let us express the dependence of the porosity and the permeability on the minimal $p_i$ and instantaneous $p$ pressures ($p > p_i$). For the point $i$ on Fig. 1, we write the dependences of the parameters of the stratum on the above-mentioned pressures

$$
m_i = m_0 \exp [a_{m0}(p - p_i)], 
$$

$$
m_i = m_0 \exp [a_{m0}(p - p_i)], 
$$

$$
k_i = k_0 \exp [a_{k0}(p - p_i)], 
$$

$$
k_i = k_0 \exp [a_{k0}(p - p_i)]. 
$$

From (2.2) we finally have expressions for determining the values of the porosity and the permeability in an element of a stratum in which the pressure is lower than the instantaneous pressure

$$
m = m_0 \exp \{a_m(p - p_0) \exp [a_{m0}(p - p_0)] \}, \quad p_i \geq p \geq p_i. 
$$

$$
k = k_0 \exp \{a_k(p - p_0) \exp [a_{k0}(p - p_0)] \} \exp [a_{k0}(p - p_0)]. 
$$

The coefficients $a_{ml}$ and $a_{kl}$ depend on the absolute value of the pressure $p_i$, from which the regeneration of the pressure starts. As a characteristic of this dependence, on the basis of the experimental data of which a review is given in [6], we introduce

$$
a_m = m_0 \exp [\eta_m(p - p_0)], 
$$

$$
a_k = k_0 \exp [\eta_k(p - p_0)], \quad p_i \geq p. 
$$

Here $\eta_m$ and $\eta_k$ are the coefficients of the irreversible change in the porosity and the permeability.

We adopt exponential dependences of the density and viscosity of the liquid on the pressure [4–8]

$$
\rho = \rho_0 \exp [a_\rho(p - p_0)], 
$$

$$
\mu = \mu_0 \exp [a_\mu(p - p_0)]. 
$$

Here $a_\rho$, $a_\mu$ are the coefficients of the compressibility of the liquid and the change in the viscosity as a function of the pressure. On the basis of (2.3)–(2.6), we write expressions for the complex parameters needed for the subsequent investigations

$$
m \rho = m_0 \rho_0 \exp \{a_{m0} \exp [\eta_m(p - p_0)] \} \exp \{a_\rho \exp [\eta_\rho(p - p_0)] \}, \quad p_i \geq p \geq p_i. 
$$