HYDRODYNAMIC STABILITY OF BOUNDARY LAYERS WITH MASS TRANSFER

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The stability of boundary layers involving mass transfer described by a two-parameter family of Fockner-Scane (F-S) solutions is analyzed. An effective method of solving the boundary problem is proposed for the F-S equation. A determinant method is proposed for solving the stability equation. The critical values of the stability characteristics are found over a wide range of gradient and mass-transfer parameters.

There have been many papers devoted to the separate influence of flow gradient and suction (or injection) on the stability of a boundary layer. Analyses have been mainly based on a single-parameter family of F-S equations (the free results may be found in [1-3]) and Pohlhausen solutions [1, 3, 4]. The effect of suction on stability was considered, for example, in [1, 3, 5]. However, until recently the combined influence of these factors on the stability of the boundary layer had never been studied, apart from being touched upon in [6].

In this paper we shall consider the stability of automodel boundary layers, in which the profile of the dimensionless velocity \( u \) is represented by a two-parameter family of F-S solutions \( u = \Phi'(\xi, \beta, \gamma) \) where \( \beta \) is the gradient parameter, \( \gamma \) the mass-transfer parameter, and \( \xi \) a dimensionless coordinate. This approach is employed because the family of F-S solutions is a fundamental basis for plotting the velocity profiles in many cases of practical interest (arrow-shaped wing, etc.), and on the basis of this family a number of exact and approximate methods of calculating boundary layers may be constructed. The results here obtained extend the possible applications of this family of solutions. A method is proposed for solving the boundary problem associated with the F-S equation in the presence of a normal velocity component at the boundary. A determinant method is used for solving the Orr-Sommerfeld equation. Using a wide range of \( \beta \) and \( \gamma \) values, the critical values of the stability characteristics (the wave number \( \alpha_* \), the Reynolds number \( \text{Re}_* \), and the perturbation phase velocity \( C_* \)) will here be determined.

1. Velocity profile. Let us consider one important class of exact solutions of the boundary-layer equation, corresponding to external circumfluence \( u_0 = c_0x^m \) (\( x, u_0 \) are the dimensional coordinate and velocity, \( c_0 \) and \( m \) are arbitrary constants).

The velocity profiles may, in this case, be represented by a two-parameter family \( u = \Phi'(\xi, \beta, \gamma) \), where \( \Phi \) is the solution to the F-S equation [3]

\[
\begin{align*}
\Phi' &= 0, \quad \Phi'' = -2m(m+1)^{-1} \\
\Phi' &= 0, \quad \Phi'' = 0, \quad \Phi' = 1
\end{align*}
\]

(1.1) (1.2)

where \( v_0 \) is the normal velocity at the wall (\( \gamma > 0 \) - suction \( \gamma < 0 \) - injection into the boundary layer), \( \delta \) is the thickness of the layer.

The principal difficulty in finding the velocity distribution lies in solving the boundary problem (1.1), (1.2), the process being complicated by the fact that \( \delta = \delta(\beta, \gamma) \). Existing methods of calculation, graphical-analytical and differential corrections, are cumbersome and inefficient. Here we shall set out a method of

dynamic variables enabling us to reduce the boundary problem (1.1), (1.2) to a Cauchy problem with little waste of time.

For convenience we divide the field of flow into region I, containing the boundary layer \( 0 \leq \xi \leq \delta^- \), \( 0 \leq u \leq 1 - \varepsilon \), and region II, characterized by potential flow \( \delta^+ \leq \xi \leq \infty \), \( 1 - \varepsilon \leq u \leq 1 \). Here \( \varepsilon = 10^{-4} \). Let us make a change of variables in (1.1). As independent variable we take \( F = \Phi'(\xi) \) and as dependent variables \( f(F) = \Phi(\xi) \) and \( \chi(F) = \Phi''(\xi) \).

Equation (1.1) for the functions \( f \) and \( \chi \) in variables \( F \) becomes

\[
\frac{df}{dF} = F \chi^{-1}, \quad \frac{d\chi}{dF} = -\chi^{-1} \left[ F \beta (F^2 - 1) \right]
\]

System (1.3) may be integrated in the range \( 0 \leq F \leq F_k, F_k = 1 - \varepsilon \) for all values of \( \beta \) and \( \gamma \). It should be noted that this method only allows us to calculate flows without any bends (inflections) in the velocity profile, since \( \chi = 0 \) is a singular point of system (1.3).

Let us formulate the boundary conditions for (1.3). In region II we express function \( \Phi \) and its derivatives in the form

\[
\Phi(\xi) = b + \xi + g(\xi), \quad \Phi'(\xi) = 1 + g'(\xi), \quad \Phi''(\xi) = g''(\xi)
\]

Here \( b \) is an arbitrary constant, the function \( g \) being chosen so that

\[
g(\xi) \to 0, \quad g'(\xi) \to 0, \quad g''(\xi) \to 0 \quad \text{as} \quad \xi \to \infty
\]

Hence for sufficiently large \( \xi \) to a fair accuracy \( \Phi(\xi) = \xi \).

Under these assumptions Eq. (1.1) may be linearized and expressed in the form

\[
g''(\xi) = 2\beta g'(\xi) - \xi g(\xi)
\]

At the point \( \xi = \delta \) we require that the following conditions should be satisfied

\[
g'(\delta^-) = g'(\delta^+), \quad g''(\delta^-) = g''(\delta^+), \quad z(\delta^-) = z(\delta^+), \quad z = g'/g''
\]

For the function \( z \) Eq. (1.5) may be written in the form

\[
z' = 1 + \xi z - 2\beta z^2
\]

The solution of (1.6) may be found both by direct integration and by expansion in series.