In a magnetohydrodynamic approximation, an investigation is made of the propagation of waves in a plasma, whose characteristic frequency \( \omega \) is much less than the collision frequency of the electrons \( \tau_e^{-1} \). It is assumed that the magnetic field is sufficiently strong so that the equality \( \omega_e \tau_e \gg 1 \) will be satisfied, where \( \omega_e \) is the cyclotron frequency of the rotation of the electrons. With large magnetic Reynolds numbers (\( R_m \gg 1 \)), which are characteristic for many astrophysical problems, this latter condition leads to a need to take account of dispersion effects connected with Hall currents, in the absence of Joule dissipation. The dispersion equation for the propagation of small perturbations is analyzed in the limiting cases of weak dispersion and of a wave propagating along the magnetic field. In the case of weak dispersion, an equation is derived for nonlinear waves. The solutions are found in the form of stationary solitons. The region of such solutions is analyzed. A typical example of a medium with Hall dispersion is an interplanetary plasma, in which the parameter \( \omega_e \tau_e \) is generally great.

**Fundamental Equations**

In a rarefied plasma or in a very strong magnetic field, the cyclotron frequency of the rotation of the electrons is usually greater than or on the same order of magnitude as the frequency of their collisions (\( \omega_e \tau_e \gg 1 \)). In such a plasma the Hall currents can be considerable, and the generalized Ohm's law will have the form

\[
\frac{j}{\sigma} = E + \frac{1}{c} \mathbf{v} \times \mathbf{B} + \frac{1}{en_e} \left( \nabla p_e - \frac{1}{e} \mathbf{j} \times \mathbf{B} \right)
\]

Here \( j \) is the current density; \( \sigma \) is the electrical conductivity; \( v \) is the velocity of the plasma; \( e, n_e, p_e \) are the values of the charge, the concentration, and the pressure of the electrons, respectively; \( c \) is the velocity of light; \( E \) is the intensity of the electrical field; \( \mathbf{B} \) is the vector of the magnetic induction. The last term in (1.1) represents the Hall currents. Eliminating \( E \) and \( j \) from (1.1) using the Maxwell equations, and neglecting displacement currents, we obtain

\[
\frac{\partial \mathbf{B}}{\partial t} = \text{rot} (\mathbf{v} \times \mathbf{B}) + \frac{cm_i}{4\pi \sigma \rho} \left( \frac{c}{en_e} \nabla \rho \right) - \frac{1}{\rho} \frac{1}{\rho} \left( \nabla p_e \right) \nabla \rho - \frac{1}{\rho} \frac{1}{\rho} \left( \nabla p_e \right) \nabla \rho - \frac{1}{\rho} \frac{1}{\rho} \left( \nabla p_e \right) \nabla \rho
\]

where \( \rho \) is the density of the plasma; \( m_i \) is the mass of an ion. The next to the last term on the right-hand side can be omitted either in the case of a weakly ionized plasma, or under the assumption that the gradients of the pressure and the concentration are parallel. We shall assume that one of these assumptions is satisfied. In addition, we assume that the magnetic Reynolds number is great (\( R_m = 4\pi \rho \nabla \mathbf{V} / c^2 \gg 1 \), where \( L \) is the characteristic dimension of the problem; \( V \) is the characteristic velocity). The characteristic velocity can be the velocity of a wave, and the characteristic length can be the length of the dispersion \( L \sim c m_i / 4\pi \rho \rho V \). With \( R_m \gg 1 \), the last term can be neglected in comparison with the first term on the right-hand side of (1.2), i.e., Joule dissipation can be neglected.
However, the dispersion terms in square brackets can be found to be considerable, since their order of magnitude is determined by the ratio $\omega_e^2 R_m$. In particular, in the solar corona this ratio may be on the order of unity.

Finally, taking account of Hall currents, the equation of the induction of the magnetic field is written in the form

$$\frac{\partial B}{\partial t} = \text{rot}(v \times B) + \frac{cm}{4\pi \rho} \left[ (\text{rot} B \cdot \nabla) B - (B \cdot \nabla) \text{rot} B + \frac{1}{\rho} (\nabla \cdot B) \text{rot} B - \frac{1}{\rho} (\nabla \cdot \text{rot} B) B \right]$$

(1.3)

Equation (1.3), together with the equations of continuity, motion, and the adiabatic law, makes up a system of equations of nondissipative magnetic hydrodynamics taking account of Hall currents, which serves to determine $B$, $v$, $\rho$ and the pressure of the plasma $p$ ($\gamma$ is the adiabatic index).

If the diffusion terms in (1.3) are neglected, the system of equations of ideal magnetic hydrodynamics is obtained [1].

The presence of dispersion effects, competing with nonlinearity, can promote the formation of stationary combined waves, i.e., solitons. In [2], a detailed analysis of such solutions is given, as well as examples of dispersion media, i.e., "small" water, ionic sound, a Hall plasma in a magnetic field, and others.

Below an investigation is made of the propagation of waves in a plasma with Hall dispersion; the equation of the induction is taken in the form (1.3). We assume that all the parameters of the plasma depend only on the single coordinate $x$ and the time $t$. Then, the system of equations describing the motion of the plasma, in projections on the coordinate axes, is written in the form

$$\begin{align*}
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} \right) &= -\frac{\partial}{\partial x} \left( \rho + \frac{B^2}{2\mu_0} \right), \\
\rho \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} \right) &= \frac{B_x}{4\pi} \frac{\partial B_x}{\partial x}, \\
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} \right) &= \frac{B_x}{4\pi} \frac{\partial B_y}{\partial x}, \\
\rho \left( \frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x} \right) &= \frac{\gamma \rho}{\gamma - 1} \left( \frac{\partial B_x}{\partial t} + \frac{\partial B_y}{\partial t} \right) = 0, \quad B_x = \text{const} \\
\frac{\partial B_x}{\partial t} &= -\frac{\partial}{\partial x} (u B_x - v B_y) - \frac{cm B_x}{4\pi \rho} \left( \frac{\partial^2 B_x}{\partial x^2} + \frac{1}{\gamma - 1} \frac{\partial B_x}{\partial x} \frac{\partial p}{\partial x} \right), \\
\frac{\partial B_y}{\partial t} &= -\frac{\partial}{\partial x} (u B_y - v B_x) - \frac{cm B_y}{4\pi \rho} \left( \frac{\partial^2 B_y}{\partial x^2} + \frac{1}{\gamma - 1} \frac{\partial B_y}{\partial x} \frac{\partial p}{\partial x} \right)
\end{align*}$$

(1.4)

Here $v$ and $w$ are the projections of the velocity on the $y$ and $z$ axes. In what follows, Eqs. (1.4) will be used for investigating the propagation of waves in a plasma.

2. Propagation of Small Perturbations

We represent the solution of the system of equations (1.4) in the form

$$\begin{align*}
p &= p_0 + p^* (x, t), \\
\rho &= \rho_0 + \rho^* (x, t), \\
B &= B_0 + b(x, t), \\
v &= v_0 + v^* (x, t)
\end{align*}$$

(2.1)

Here the subscript 0 relates to constant parameters in an unperturbed plasma, and $p^*, \rho^*, b, v^*$ are small perturbations of the pressure, the density, the magnetic field, and the velocity. Substituting (2.1) into (1.4), by linearizing the system of equations obtained with respect to small parameters, and seeking the solution in the form

$$f = f^0 \exp \left[ i (\omega t - kx) \right]$$

(2.2)

where $f^0$ is the amplitude of a perturbation, and $f$ is understood as any one of the sought functions, we obtain the following homogeneous system of algebraic equations for the amplitudes of the perturbations:

$$\rho_0 \lambda u = \rho + \frac{1}{4\pi} (B_{0v} b_x + B_{0x} b_v), \quad \lambda \rho = \rho_0 u$$

422